

Introduction to Multilevel Models for Longitudinal and Repeated Measures Data

- Today's Class:
 - Features of longitudinal data
 - Features of longitudinal models
 - What can MLM do for you?
 - What to expect in this course (and the next course)

What is CL(D)P 944 about?

- “Longitudinal” data
 - Same individual units of analysis measured at different occasions (which can range from milliseconds to decades)
- “Repeated measures” data (*if time permits*)
 - Same individual units of analysis measured via different items, using different stimuli, or under different conditions
- Both of these fall under a more general category of “multivariate” data of varying complexity
 - The link between them is the use of **random effects** to describe covariance of outcomes from the same unit

Data Requirements for Our Models

- A useful outcome variable:
 - Has an interval scale*
 - A one-unit difference means the same thing across all scale points
 - In subscales, each contributing item has an equivalent scale
 - **Other kinds of outcomes will be analyzed using generalized multilevel models instead, but estimation will be more challenging*
 - Has scores with the same meaning over observations
 - Includes meaning of construct
 - Includes how items relate to the scale
 - Implies measurement invariance
- FANCY MODELS CANNOT SAVE BADLY MEASURED VARIABLES OR CONFOUNDED RESEARCH DESIGNS.

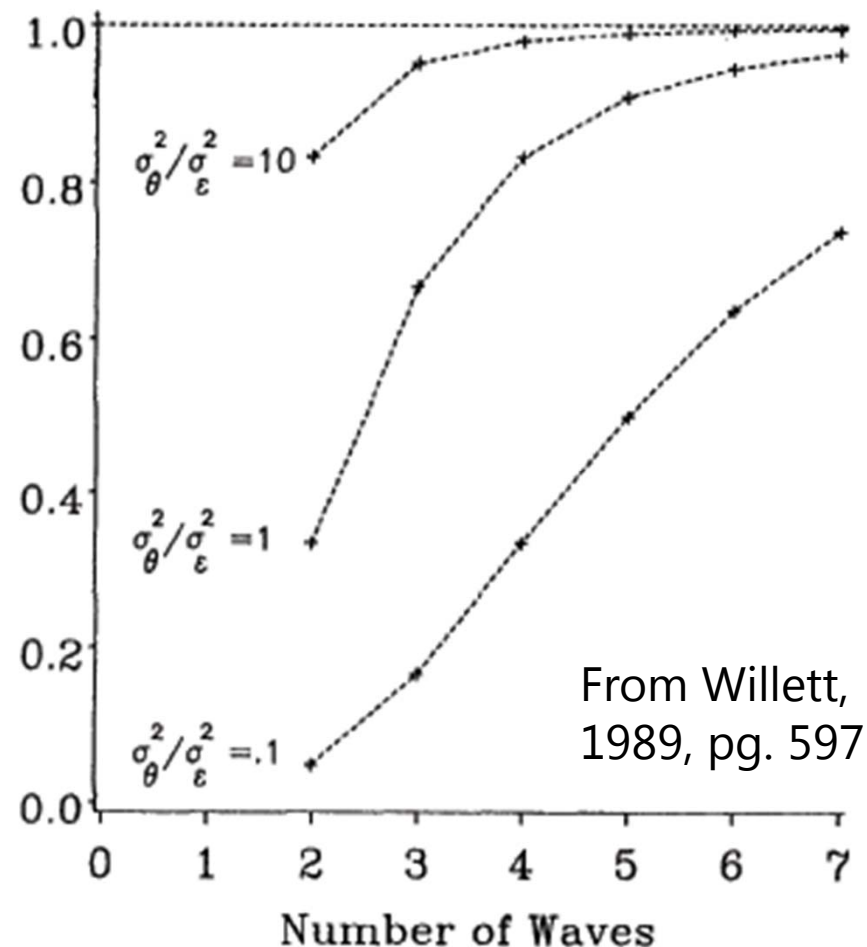
Requirements for Longitudinal Data

- Multiple OUTCOMES from same sampling unit (person)
 - 2 is the minimum, but just 2 can lead to problems:
 - Only 1 kind of change is observable (1 difference)
 - Can't distinguish "real" individual differences in change from error
 - Repeated measures ANOVA is just fine for 2 observations
 - Necessary assumption of "sphericity" is satisfied with only 2 observations even if compound symmetry doesn't hold
 - More data is better (with diminishing returns)
 - More occasions → better description of the form of change
 - More persons → better estimates of amount of individual differences in change; better prediction of those individual differences
 - More items/stimuli/groups → more power to show effects of differences between items/stimuli/groups

Power in Longitudinal Data

- More occasions are better!
 - Can examine more complex growth functions
 - Can get more reliable individual growth parameters
- More people are better!
 - Can get more reliable measures of individual differences

Reliability of Slopes by Signal to Noise Ratio and # Occasions



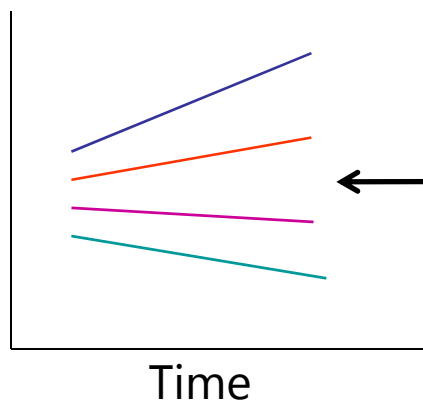
Levels of Analysis in Longitudinal Data

- Between-Person (BP) Variation:
 - **Level-2** – “**INTER**-individual Differences” – Time-Invariant
 - All longitudinal studies begin as cross-sectional studies
- Within-Person (WP) Variation:
 - **Level-1** – “**INTRA**-individual Differences” – Time-Varying
 - Only longitudinal studies can provide this extra information
- Longitudinal studies allow examination of both types of relationships simultaneously (and their interactions)
 - Any variable measured over time usually has both BP and WP variation
 - BP = more/less than other people; WP = more/less than one's average
- I use “person” here, but level-2 units can be anything that is measured repeatedly (like animals, schools, countries...)

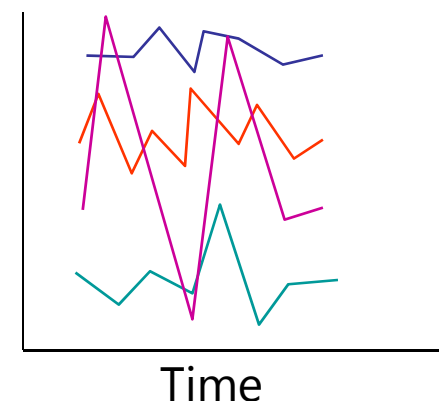
A Longitudinal Data Continuum

- **Within-Person Change:** Systematic change
 - Magnitude or direction of change can be different across individuals
 - “Growth curve models” → Time is meaningfully sampled
- **Within-Person Fluctuation:** No systematic change
 - Outcome just varies/fluctuates over time (e.g., emotion, stress)
 - Time is just a way to get lots of data per individual

Pure WP Change



Pure WP Fluctuation



What is a Multilevel Model (MLM)?

- Same as other terms you have heard of:
 - **General Linear Mixed Model** (if you are from statistics)
 - *Mixed* = Fixed and Random effects
 - **Random Coefficients Model** (also if you are from statistics)
 - Random coefficients = Random effects = latent variables/factors
 - **Hierarchical Linear Model** (if you are from education)
 - Not the same as hierarchical regression
- Special cases of MLM:
 - Random Effects ANOVA or Repeated Measures ANOVA
 - (Latent) Growth Curve Model (where “Latent” implies SEM)
 - Within-Person Fluctuation Model (e.g., for daily diary data)
 - Clustered/Nested Observations Model (e.g., for kids in schools)
 - Cross-Classified Models (e.g., “value-added” models)
 - Psychometric Models (e.g., factor analysis, item response theory)

The Two Sides of Any Model

- **Model for the Means:**

- *Aka* **Fixed Effects**, Structural Part of Model
- What you are used to **caring about for testing hypotheses**
- How the expected outcome for a given observation varies as a function of values on predictor variables

- **Model for the Variance:**

- *Aka* **Random Effects and Residuals**, Stochastic Part of Model
- What you are used to **making assumptions about** instead
- How residuals are distributed and related across observations (persons, groups, time, etc.) → these relationships are called “dependency” and ***this is the primary way that multilevel models differ from general linear models (e.g., regression)***

Dimensions for Organizing Models

- Outcome type: General (normal) vs. Generalized (not normal)
- Dimensions of sampling: One (so one variance term per outcome) vs. **Multiple** (so multiple variance terms per outcome) → **OUR WORLD**
- **General Linear Models**: conditionally normal outcome distribution, **fixed effects** (identity link; only one dimension of sampling)
- **Generalized Linear Models**: **any conditional outcome distribution**, **fixed** effects through **link functions**, no random effects (one dimension)
- **General Linear Mixed Models**: conditionally normal outcome distribution, **fixed and random effects** (identity link, but multiple sampling dimensions)
- **Generalized Linear Mixed Models**: **any conditional outcome distribution**, **fixed and random effects** through **link functions** (multiple dimensions)
 - Same concepts as for this course, but with more complexity in estimation
- “Linear” means fixed effects predict the *link-transformed conditional mean* of DV in a linear combination of (effect*predictor) + (effect*predictor)...

Note: Least Squares is only for GLM

Options for Longitudinal Models

- Although models and software are logically separate, longitudinal data can be analyzed via multiple analytic frameworks:
 - “Multilevel/Mixed Models”
 - Dependency over time, persons, groups, etc. is modeled via random effects (multivariate through “levels” using stacked/long data)
 - Builds on GLM, generalizes easier to additional levels of analysis
 - “Structural Equation Models”
 - Dependency over time *only* is modeled via latent variables (single-level analysis using multivariate/wide data)
 - Generalizes easier to broader analysis of latent constructs, mediation
 - Because random effects and latent variables are the same thing, many longitudinal models can be specified/estimated either way
 - And now “Multilevel Structural Equation Models” can do it all (maybe)...

What can MLM do for you?

- 1. Model dependency across observations**
 - Longitudinal, clustered, and/or cross-classified data? No problem!
 - Tailor your model of sources of correlation to your data
- 2. Include categorical or continuous predictors at any level**
 - Time-varying, person-level, group-level predictors for each variance
 - Explore reasons for dependency, don't just control for dependency
- 3. Does not require same data structure for each person**
 - Unbalanced or missing data? No problem!
- 4. You already know how (or you will soon)!**
 - Use SPSS Mixed, **SAS Mixed**, Stata, Mplus, R, HLM, MlwiN...
 - What's an intercept? What's a slope? What's a pile of variance?

1. Model Dependency

- Sources of dependency depend on the sources of **variation** created by your sampling design: residuals for outcomes from the same unit are likely to be related, which violates the GLM “independence” assumption
- **“Levels” for dependency** = “levels of random effects”
 - Sampling dimensions can be **nested**
 - e.g., time within person, time within group, trial within person
 - If you can't figure out the direction of your nesting structure, odds are good you have a **crossed sampling design** instead
 - e.g., persons crossed with items, raters crossed with targets
 - To have a “level”, there must be random outcome variation due to sampling that **remains** after including the model's fixed effects
 - e.g., treatment vs. control does not create another level of “group” (but it would if you had multiple treatment and multiple control groups)

Longitudinal dependency comes from...

- Mean differences across sampling units (e.g., persons)
 - Creates *constant* dependency over time
 - Will be represented by a random intercept in our models
- Individual differences in effects of predictors
 - Individual differences in change over time, stress reactivity
 - Creates non-constant dependency, the size of which depends on the value of the predictor at each occasion
 - Will be represented by random slopes in our models
- Non-constant within-person correlation for unknown reasons (time-dependent autocorrelation)
 - Can add other patterns of correlation as needed for this

Why care about dependency?

- In other words, what happens if we have the wrong model for the variances (assume independence instead)?
- **Validity of the tests of the predictors** depends on having the “most right” model for the variances
 - Estimates will usually be ok → come from model for the means
 - Standard errors (and thus p -values) can be compromised
- The sources of variation that exist in your outcome will dictate **what kinds of predictors** will be useful
 - Between-Person variation needs Between-Person predictors
 - Within-Person variation needs Within-Person predictors
 - Between-whatever variation needs Between-whatever predictors...

2. Include categorical or continuous predictors at any level of analysis

- “ANOVA” test differences among discrete groups
- “Regression” tests slopes for continuous predictors
- What if a predictor is assessed repeatedly but can't be characterized by discrete “conditions”?
 - ANOVA or Regression won't work → you need MLM
- Some things don't change over time → time-invariant
- Some things do change over time → time-varying
- Some things are measured at higher levels
- Interactions are possible at same level or across levels

3. Does not require same data structure per person (by accident or by design)

RM ANOVA: uses **multivariate** (wide) data structure:

ID	Sex	T1	T2	T3	T4
100	0	5	6	8	12
101	1	4	7	.	11

People missing any data are excluded (data from ID 101 are not included at all)

MLM: uses **stacked** (long) data structure:

Only rows missing data are excluded

100 uses 4 cases

101 uses 3 cases

ID	Sex	Time	Y
100	0	1	5
100	0	2	6
100	0	3	8
100	0	4	12

101	1	1	4
101	1	2	7
101	1	3	.
101	1	4	11

Time can also be **unbalanced** across people such that each person can have his or her own measurement schedule: Time "0.9" "1.4" "3.5" "4.2"...

4. You already know how!

- If you can do GLM, you can do MLM
(and if you can do generalized linear models, you can do generalized multilevel models, too)
- How do you interpret an estimate for...
 - the intercept?
 - the effect of a continuous variable?
 - the effect of a categorical variable?
 - a variance component (“pile of variance”)?

CLDP 944: This Semester's Topics

- Make Friends with SAS
- Review of single-level linear models
- Within-person analysis via ANOVA
- Describing within-person fluctuation via alternative covariance structure models
 - R matrix structures with and without a random intercept
- Describing within-person change via random effects
 - Polynomial, piecewise, and nonlinear models
- Review of interactions in single-level linear models
- Time-invariant (level-2) predictors
- If time permits, crossed random effects models for other kinds of repeated measures data (subjects by items)

CLDP 945: Next Semester's Topics

- Two-level models for persons in groups
- Time-varying predictors that show WP fluctuation
- Local-scale models for predicting heterogeneity of variance
- Multivariate models
 - For multiple outcomes
 - For time-varying predictors that show WP change
 - For "multilevel SEM"
- Models for accelerated longitudinal data
- Cross-classified models for persons in time-varying groups or other kinds of repeated measures designs (subjects by items)
- Three-level models
 - For measurement burst designs
 - For persons in time-invariant groups