

**Systematically Varying Effects  
Created by Cross-Level  
Interactions in Multilevel Models  
are Indeed Permissible**

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at the Annual Meeting of the Society of  
Multivariate Experimental Psychology, 10/6/2017**

# Background: 2011

## Systematically Varying Effects in Multilevel Models: Permissible or Problematic?

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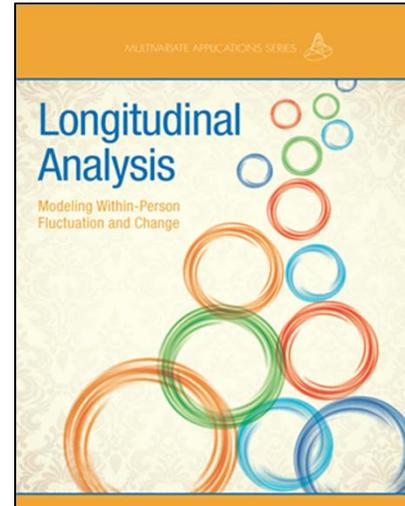
Presented in Norman, Oklahoma at the Annual Meeting of the  
Society of Multivariate Experimental Psychology, 10/15/2011

# Six Years of Distraction...

**Bought and sold  
in Lincoln, NE**



**Huey Hofflin:  
kept alive for  
2.5 years now!**



**Bought and remodeled  
in Lawrence, KS**



**Finally  
finished and  
for sale!**



# The Issue: A Longitudinal Example

$$\text{Level-1 model: } y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + e_{ti}$$

This level-2 model is ok...

$$\begin{aligned}\beta_{0i} &= \gamma_{00} + \gamma_{01}(\text{Group}_i) + U_{0i} \\ \beta_{1i} &= \gamma_{10} + \gamma_{11}(\text{Group}_i) + U_{1i}\end{aligned}$$

But is this level-2 model ok?

$$\begin{aligned}\beta_{0i} &= \gamma_{00} + \gamma_{01}(\text{Group}_i) + U_{0i} \\ \beta_{1i} &= \gamma_{10} + \gamma_{11}(\text{Group}_i) \quad \square\end{aligned}$$

“Conditionally  
Random”

“Systematically  
Varying”

“Fixed”

*Complexity Continuum of Level-1 Effect of Time*

# Testing Cross-Level Interactions

If the level-1 effect is conditionally random:

$$SE_{\gamma_{11}}^2 = \frac{(L1n * \tau_{U_1}^2 * \sigma_{L1}^2) + \sigma_e^2}{L2n * L1n * \sigma_c^2}$$

Denominator DF will resemble **level-2**  
=  $L2n - L2k$

*"A significant random slope is a precursor..."*

If the level-1 effect is systematically varying:

$$SE_{\gamma_{11}}^2 = \frac{\sigma_e^2}{L2n * L1n * \sigma_c^2}$$

Denominator DF will resemble **level-1**  
=  $(L2n * L1n) - L2n - L1k$

*"What's the point?"*

# Simulation Research Question

**Do cross-level interactions that create systematically varying effects have inflated Type I error rates in small samples (due to more likely Type II errors for  $\tau_{U_1}^2$ )?**

# Simulation Design: 10,000 reps

$\gamma_{11} = 0$  always

L2n = 10–50 by 5

L1n = 3–10 by 1

Conditionally Random Slope Model:

Level-1  $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + e_{ti}$

Int:  $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Group}_i) + U_{0i}$

Time:  $\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Group}_i) + U_{1i}$

Intraclass Correlation

ICC = .04 to .50 by .02

$$\text{ICC} = \frac{\tau_{U_0}^2}{\tau_{U_0}^2 + \sigma_e^2}$$

Slope Reliability

SR = .04 to .50 by .02

$$\text{SR} = \frac{\tau_{U_1}^2}{\tau_{U_1}^2 + \frac{\sigma_e^2}{L1n * \sigma_{L1}^2}}$$

$\tau_{U_1}^2 > 0?$   $-2\Delta LL > 5.14$

# Simulation Results

First:

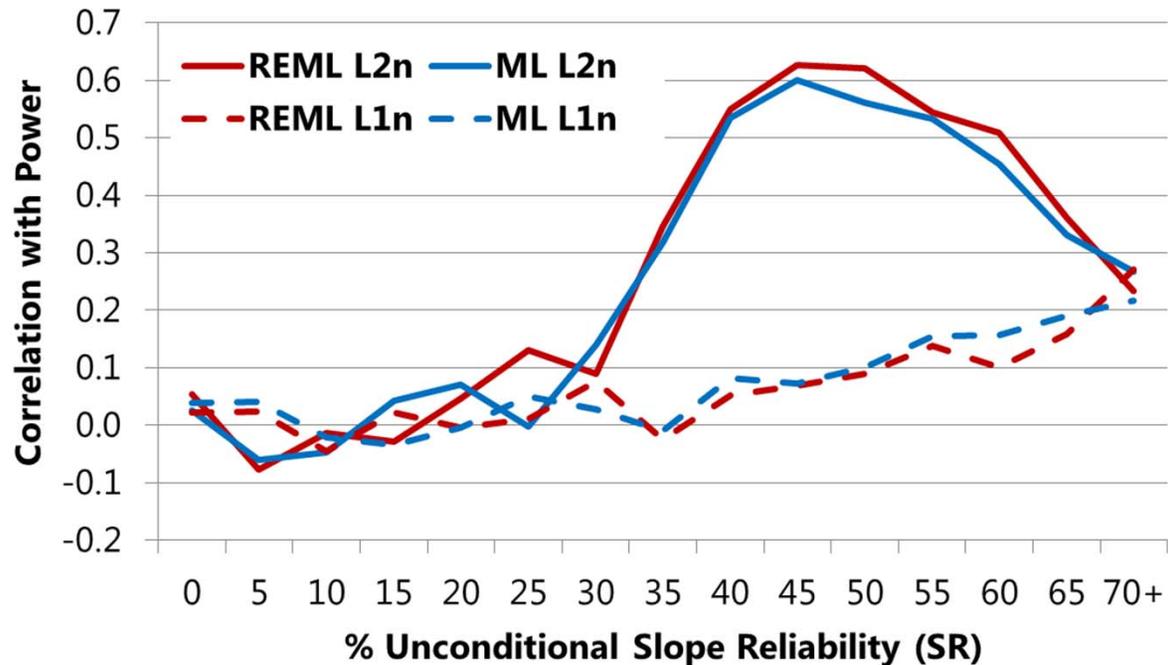
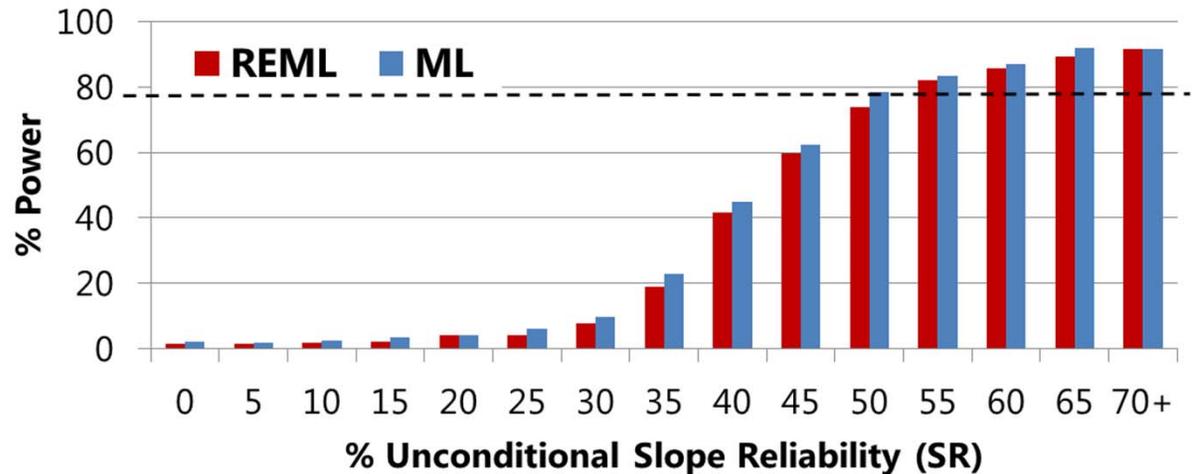
What is the chance of making a Type II error for the random slope  $\tau_{U_1}^2$ ?

# Power for Random Slope Variance $\tau_{U_1}^2$

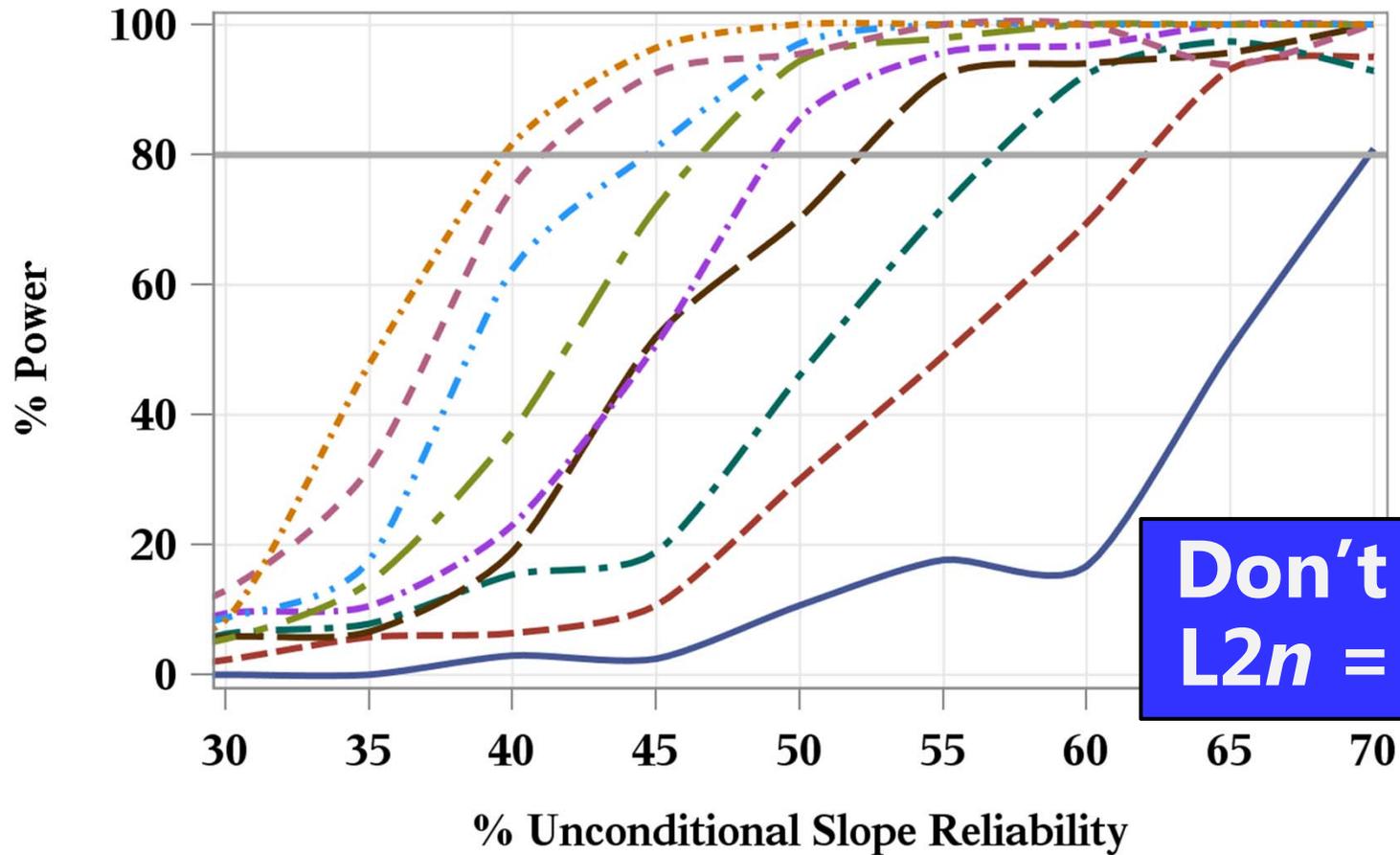
**Slope Reliability mattered most**

**L2n helped for mid-range SR**

**L1n helped only for high SR**



# Power for Random Slope Variance $\tau_{U_1}^2$

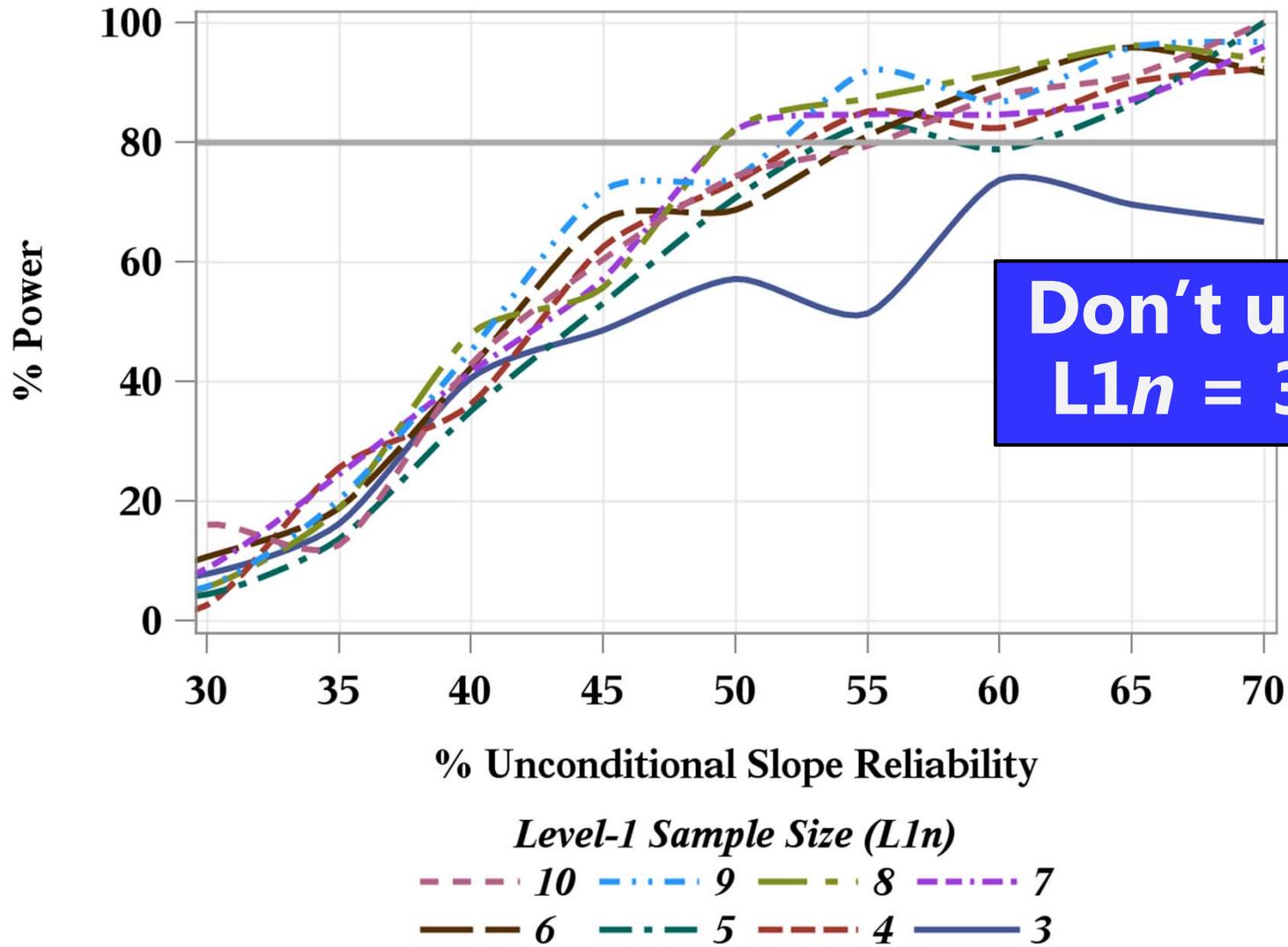


**Don't use  
L2n = 10!**

*Level-2 Sample Size (L2n)*

..... 50	..... 45	..... 40	..... 35	..... 30
..... 25	..... 20	..... 15	..... 10	

# Power for Random Slope Variance $\tau_{U_1}^2$



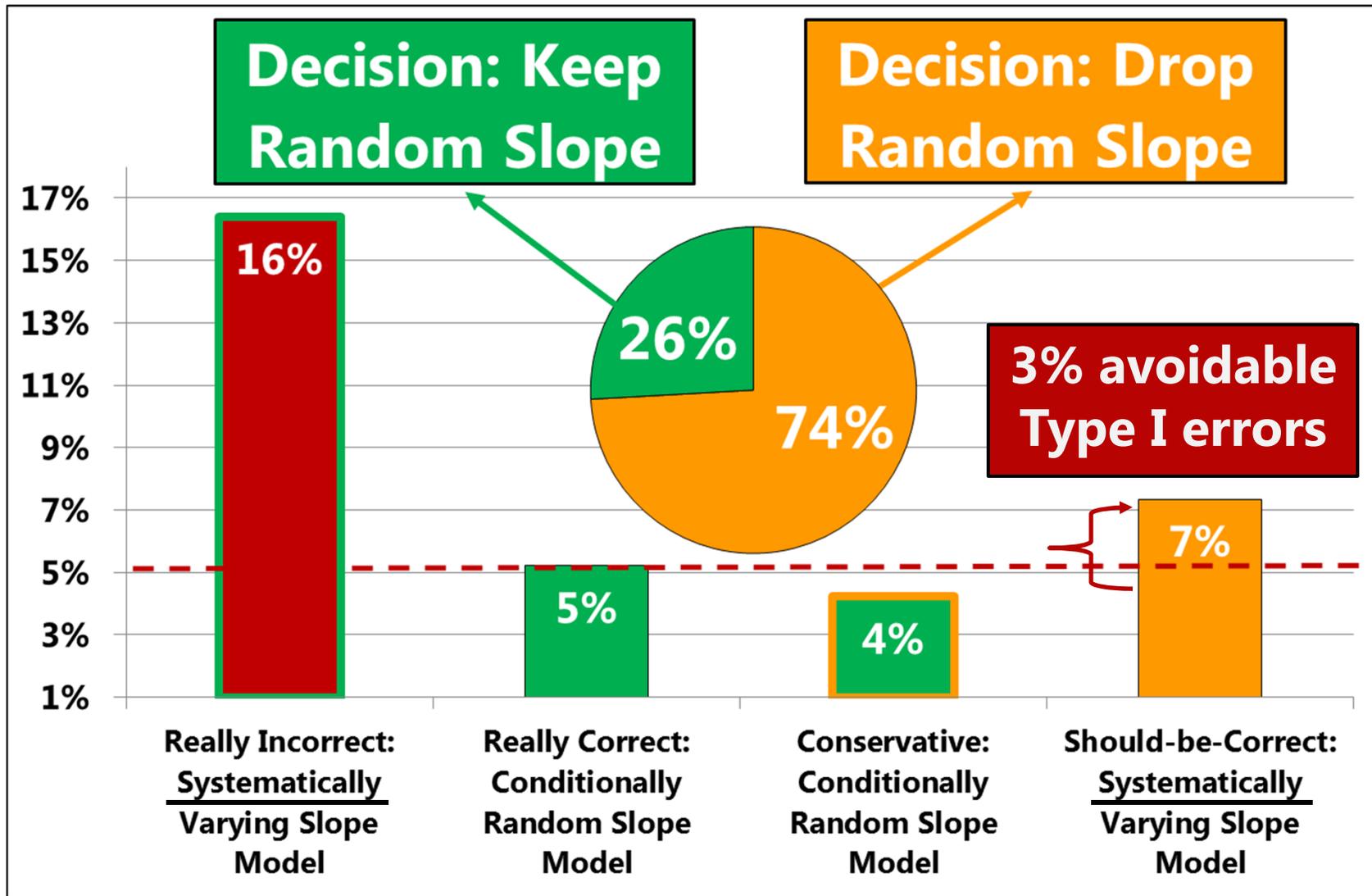
# Simulation Results

## Second:

**How often do we make a Type I error for a cross-level interaction given the decision to keep or drop the random slope?**

**Best-case scenario:  
REML Estimation with  
Kenward-Roger DDF**

# Cross-Level Interaction Type I Errors



# Simulation Results

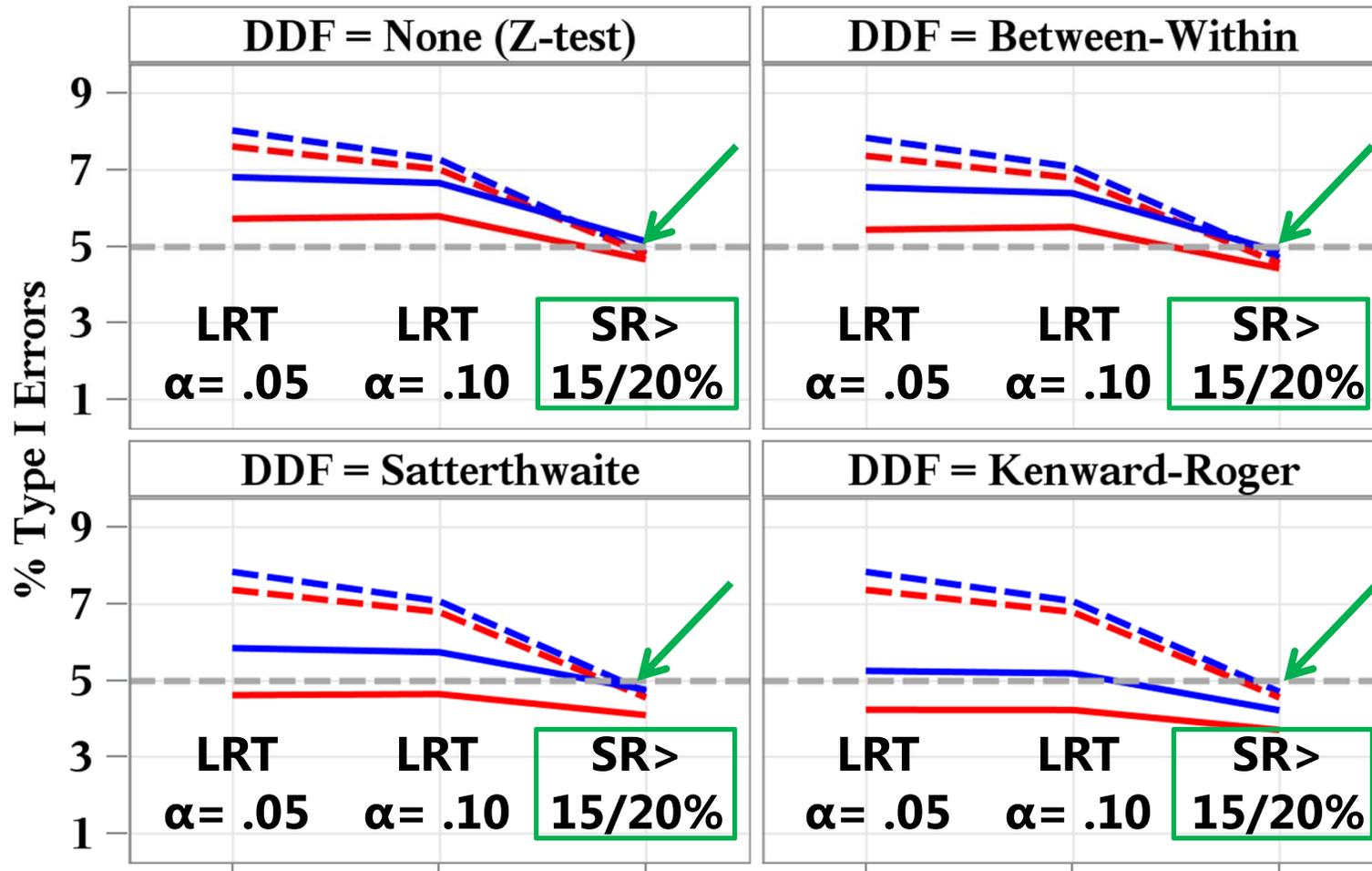
## Third:

**How can we avoid Type I errors that result from incorrectly choosing the systematically varying slope model (when  $\tau_{U_1}^2$  should have been kept)?**

**Reduce Type II errors for  $\tau_{U_1}^2$   
by using slope reliability  
for conditional model selection**

# Reps with Decision to Drop $\tau_{U_1}^2$

- Systematically Varying Slope Model in ML
- Systematically Varying Slope Model in REML
- Conditionally Random Slope Model in ML
- Conditionally Random Slope Model in REML



# Summary and Recommendations

Systematically varying effects are **PERMISSIBLE** IF  $U_{1i}$  is *really* not there.

This level-2 model can be ok.

$$\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Group}_i) + U_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Group}_i)$$



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**Given due diligence, maximal models are *not* always needed for testing cross-level interactions:**

1. Keep any **unconditional** random slope variance that is significant using a traditional LRT OR with unconditional **slope reliability >20% in REML or 15% in ML.**
2. Keep any **conditional** random slope variance that remains estimable—only when it is 0 can it be safely removed.

**Questions or comments?**

**Email Lesa Hoffman:  
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**Slides available at:  
[www.LesaHoffman.com](http://www.LesaHoffman.com)**