Systematically Varying Effects in Multilevel Models: Permissible or Problematic?

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The Issue

Given this Level 1 model: $Height_{ti} = \beta_{0i} + \beta_{1i}(Time_{ti}) + e_{ti}$

This level-2 model is ok…	But is this level-2 model ok?
$\beta_{0i} = \gamma_{00} + \gamma_{01}(Gender_i) + U_{0i}$	$\beta_{0i} = \gamma_{00} + \gamma_{01}(Gender_i) + U_{0i}$
$\beta_{1i} = \gamma_{10} + \gamma_{11}(Gender_i) + U_{1i}$	$\beta_{1i} = \gamma_{10} + \gamma_{11}(Gender_i)$

"Random"

"Systematically Varying"

"Fixed"

Complexity Continuum of Level-1 Effects

Systematically Varying Effects…

Are PERMISSIBLE because:

- Fixed effects have more power than random slope variances, so cross-level interactions like $\gamma_{11}(\text{Gender}_i)(\text{Time}_{ti})$ could be significant even *without* a significant random Time_{ti} slope variance
- May happen if *all* random slope variance is explained (good!)

Are PROBLEMATIC because:

- Without a random Time_{ti} slope variance, the cross-level interaction of $\gamma_{11}(\text{Gender}_i)(\text{Time}_{ti})$ would be tested using a different SE and with level-1 instead of level-2 denominator degrees of freedom
- What's the point? (bad!)

Simulation

Design Conditions…

- # Level-1 units: *5, 30*
- # Level-2 units: *20, 50, 100*
- Balanced: *no, yes*
- Denominator DF Method: *none (Z-test), BW, Satt, KR*

…. that didn't really matter (partial $\eta^2 \leq .01$)

Analysis Outcomes

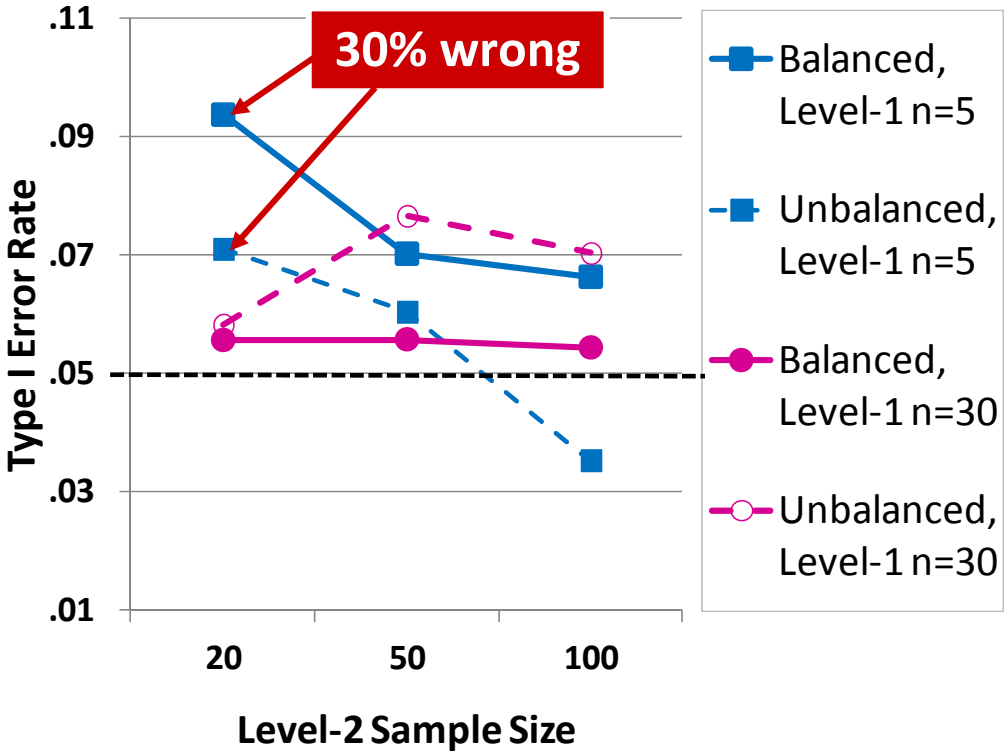
(using $-2\Delta LL > 5.14$ for $p < .05$)

Truth in Data…	**Empirical Decision** % Occurrence in Design Conditions	
Small Random Slope Variance	**Keep 3-12%**	**Remove 88-94%**
Large Random Slope Variance	**Keep 70-100%**	**Remove 0-30%**

Outcome: Type I error rate for a cross-level interaction **($\gamma_{11} \approx 0$)**

Type I Error for Cross-Level Interaction

NS or NPD random slope variance was removed…



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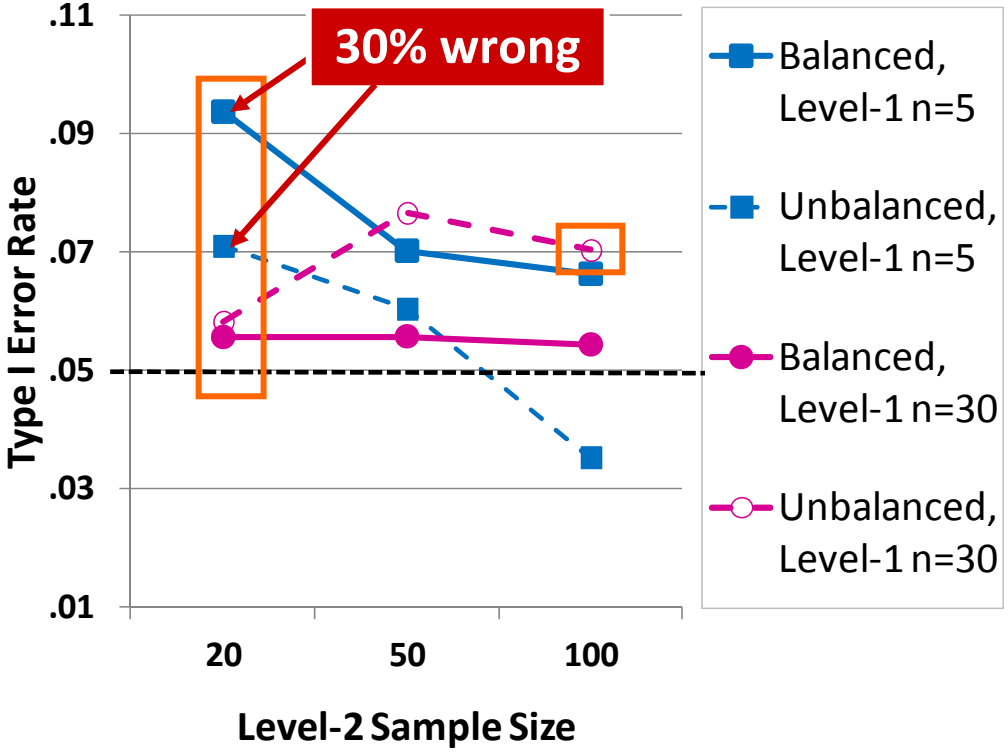
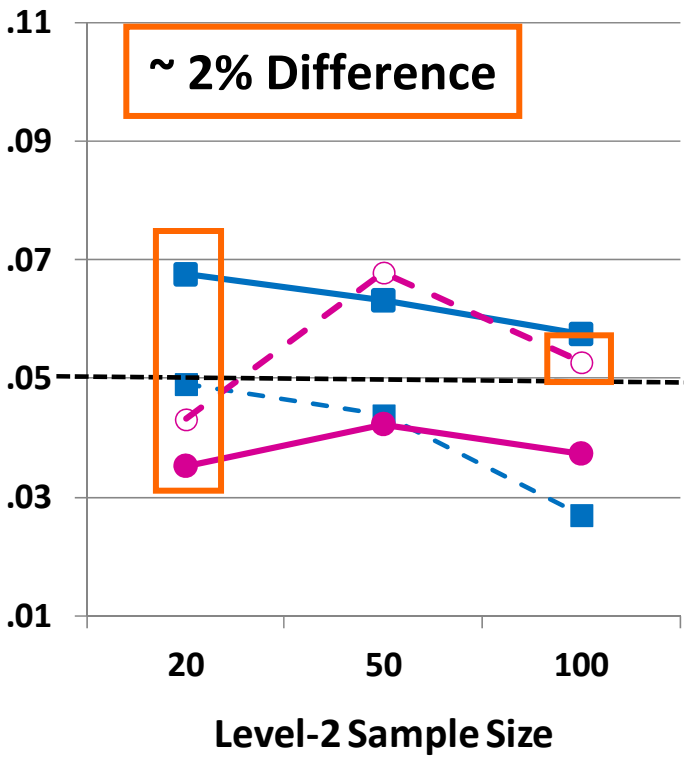
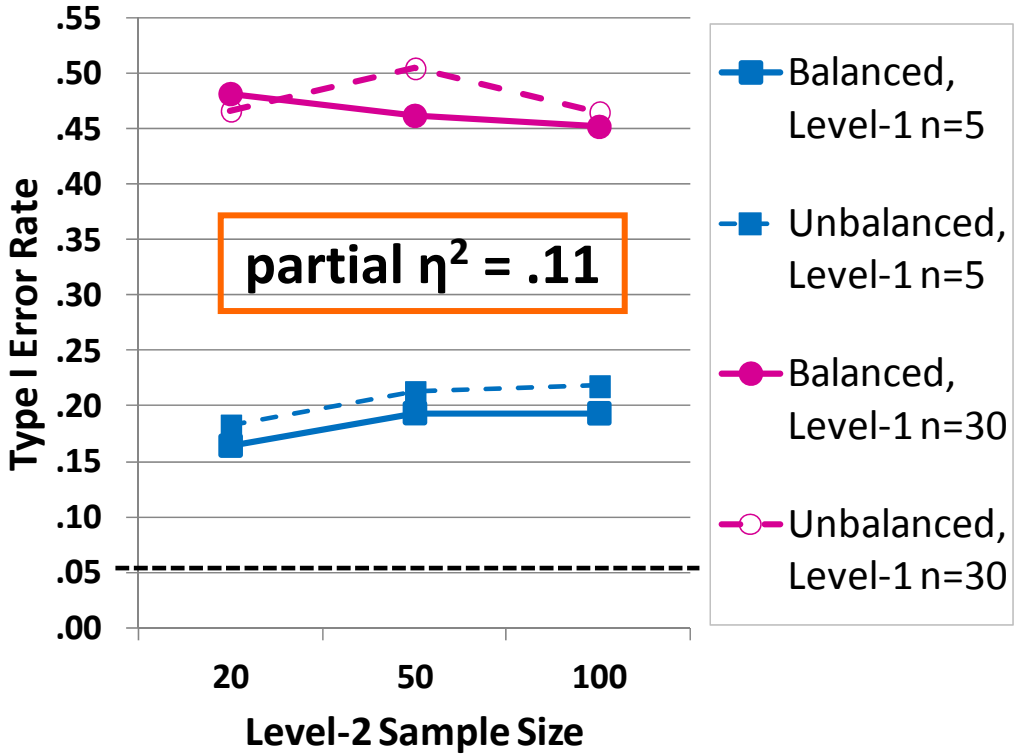


What if we *had* kept the random slope variance?



Type I Error for Cross-Level Interaction

Do NOT remove a significant random slope variance!



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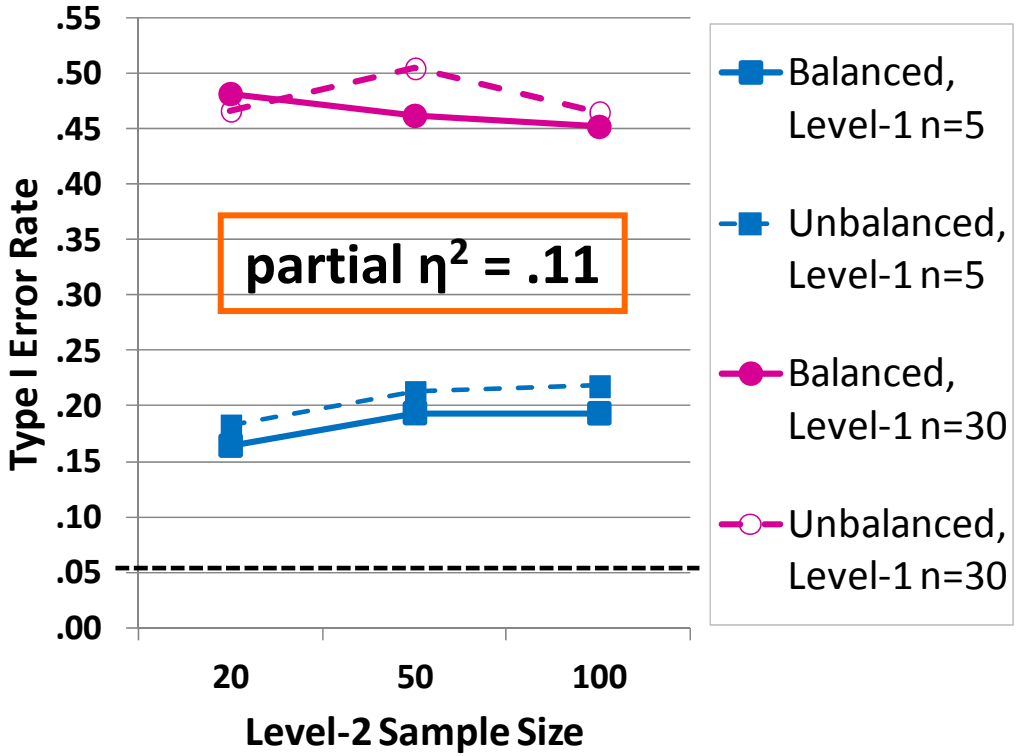

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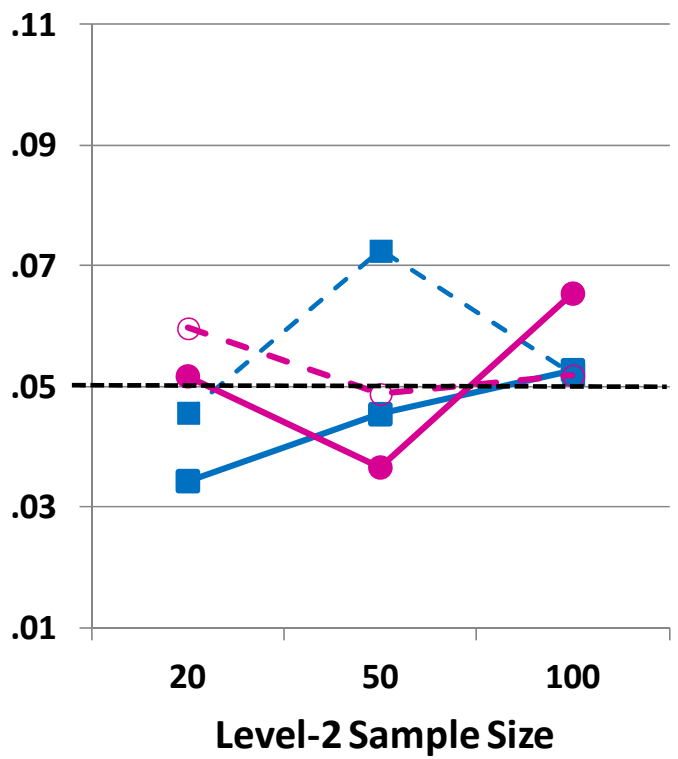
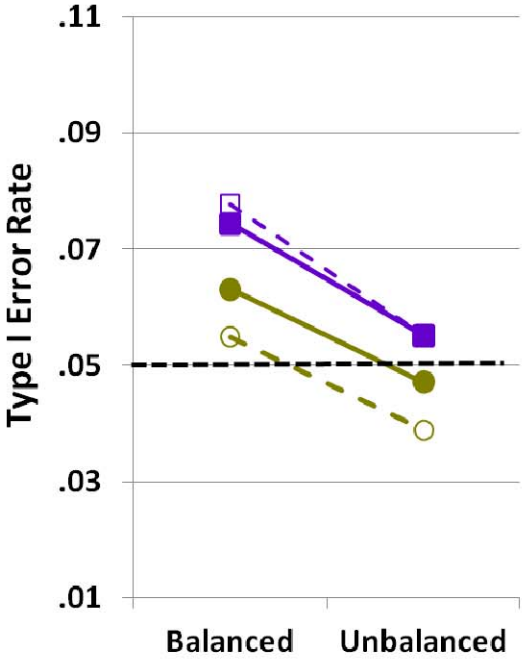
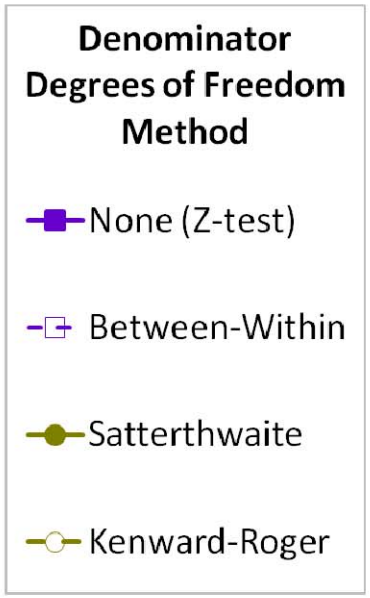


Type I Error for Cross-Level Interaction

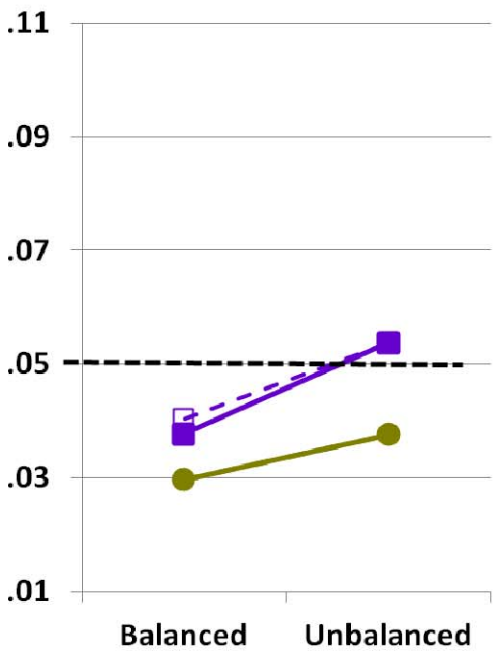
At smallest sample size: Level-2 N=20, Level-1 n=5…

Nonsignificant **random slope variance included**





Significant **random slope variance included**



… otherwise DDF method didn't matter at all

Conclusions

Level-2 model with a *systematically* varying slope:

$$\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Gender}_i) + U_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Gender}_i) \quad \boxed{?}$$

Possibly problematic when…

- Not enough power to detect the random slope variance
  - e.g., 30% wrong here if N=20, n=5; 3% wrong if N=50, n=5
  - But what can be done to fix this?

Reasonably permissible otherwise…

- Type I error ≈ 3% to 7% if the random slope is not needed

Thank you!

Questions or comments?

Email Lesa Hoffman:
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Slides available at:
http://psych.unl.edu/hoffman/Sheets/Talks.htm