

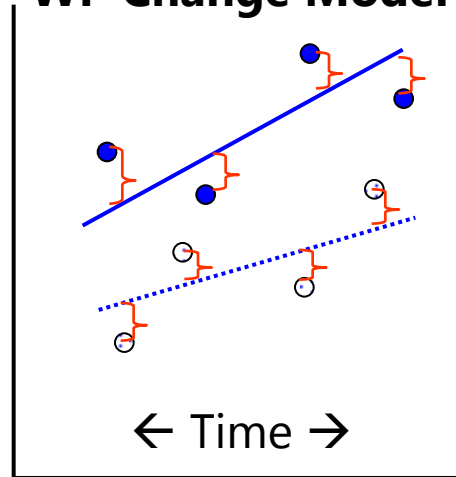
Univariate Multilevel Modeling of Time-Varying Predictors

- Topics:
 - Time-varying predictors that fluctuate over time
 - Fixed level-1 effects using person-Mean-Centering (PMC)
 - Or “Variable-Based-Centering” more broadly
 - Fixed level-1 effects using grand-Mean-Centering (GMC)
 - Or “Constant-Based-Centering” more broadly
 - Interactions and random effects when using Person-MC vs. Grand-MC

The Joy of Time-Varying Predictors

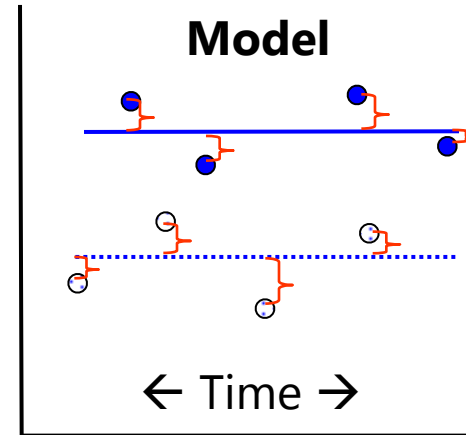
- TV predictors predict leftover **WP (residual) variation**:

WP Change Model



If model for time works, then residuals should look like this →

WP Variation Model



- Modeling time-varying predictors is complicated because they represent an **aggregated effect**:
 - Effect of the *between-person* variation in the predictor x_{ti} on Y
 - Effect of the *within-person* variation in the predictor x_{ti} on Y
 - For now we are assuming the predictor x_{ti} only **fluctuates** over time...
 - We will need a **different model** when x_{ti} changes individually over time...

The Joy of Time-Varying Predictors

- Time-varying (TV) predictors usually carry 2 kinds of effects because they are really 2 predictor variables, not 1
- Example: Stress measured daily
 - Some days are worse than others:
 - **WP variation in stress** (*represented as deviation from own mean*)
 - Some people just have more stress than others all the time:
 - **BP variation in stress** (*represented as person mean predictor over time*)
- Can quantify each source of variation with an **ICC**
 - $ICC = (BP \text{ variance}) / (BP \text{ variance} + WP \text{ variance})$
 - **ICC > 0?** TV predictor has **BP** variation (so it *could* have a BP effect)
 - **ICC < 1?** TV predictor has **WP** variation (so it *could* have a WP effect)

Between-Person vs. Within-Person Effects

- Between-person and within-person effects in SAME direction
 - Stress → Health?
 - **BP: People with more chronic stress than other people may have worse general health than people with less chronic stress**
 - **WP: People may feel worse than usual when they are currently under more stress than usual (regardless of what “usual” is)**
- Between-person and within-person effects in OPPOSITE directions
 - Exercise → Blood pressure?
 - **BP: People who exercise more often generally have lower blood pressure than people who are more sedentary**
 - **WP: During exercise, blood pressure is higher than during rest**
- Variables have different **meanings** at different levels!
- Variables have different **scales** at different levels

3 Kinds of Fixed Effects for TV Predictors

- **Is the Level-2 Between-Person (BP) effect significant?**

- Are people with higher predictor values than other people (*on average over time*) also higher on Y than other people (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random intercept variance ($\tau_{U_0}^2$)?

- **Is the Level-1 Within-Person (WP) effect significant?**

- If you have higher predictor values than usual (*at this occasion*), do you also have higher outcomes values than usual (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance (σ_e^2)?

- **Are BP and WP effects different : Is there a level-2 contextual effect?**

- After controlling for the absolute value of TV predictor at each occasion, is there still an incremental contribution from having a higher person mean of the TV predictor (i.e., does one's general tendency predict $\tau_{U_0}^2$ above and beyond)?
- If there is no contextual effect, then the BP and WP effects of the TV predictor show convergence, such that their effects are of equivalent magnitude

Modeling TV Predictors (labeled as x_{ti})

- **Level-2 effect of x_{ti} :**

- The level-2 effect of x_{ti} is usually represented by the person's mean of time-varying x_{ti} across time (labeled as **PM x_i** or \bar{X}_i)
- PMx_i should be centered at a CONSTANT (grand mean or other) so that 0 is meaningful, just like any other time-invariant predictor

- **Level-1 effect of x_{ti} can be included two different ways:**

- "**Group-mean-centering**" → "**person-mean-centering**" in longitudinal, in which level-1 predictors are centered using a level-2 VARIABLE
- "**Grand-mean-centering**" → level-1 predictors are centered using a CONSTANT (not necessarily the grand mean; it's just called that)
- Note that these 2 choices do NOT apply to the level-2 effect of x_{ti}
 - But the interpretation of the level-2 effect of x_{ti} WILL DIFFER based on which centering method you choose for the level-1 effect of x_{ti} !

Person-Mean-Centering (P-MC)

- In P-MC, we partition the TV predictor x_{ti} into 2 variables that directly represent its BP (level-2) and WP (level-1) sources of variation, and **include these variables as the predictors instead**:
- **Level-2, PM predictor = person mean of x_{ti}**
 - **$PMx_i = \bar{X}_i - C$**
 - PMx_i is centered at constant C , chosen for meaningful 0 (e.g., sample mean)
 - PMx_i is positive? Above sample mean → “more than other people”
 - PMx_i is negative? Below sample mean → “less than other people”
- **Level-1, WP predictor = deviation from person mean of x_{ti}**
 - **$WPx_{ti} = x_{ti} - \bar{X}_i$** (note: uncentered person mean \bar{X}_i is used to center x_{ti})
 - WPx_{ti} is NOT centered at a constant; is centered at a VARIABLE
 - WPx_{ti} is positive? Above your own mean → “more than usual”
 - WPx_{ti} is negative? Below your own mean → “less than usual”

Within-Person Fluctuation Model with Person-Mean-Centered Level-1 x_{ti}

→ WP and BP Effects directly through separate parameters

x_{ti} is person-mean-centered into WPx_{ti} , with PMx_i at L2:

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}(WPx_{ti}) + e_{ti}$$

$WPx_{ti} = x_{ti} - \bar{X}_i \rightarrow$ it has only Level-1 WP variation

$$\text{Level 2: } \beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + u_{0i}$$

$PMx_i = \bar{X}_i - C \rightarrow$ it has only Level-2 BP variation

$$\beta_{1i} = \gamma_{10} + (u_{1i})$$

u_{1i} is a random slope for the WP effect of x_{ti}

γ_{10} = WP main effect of having more x_{ti} than usual

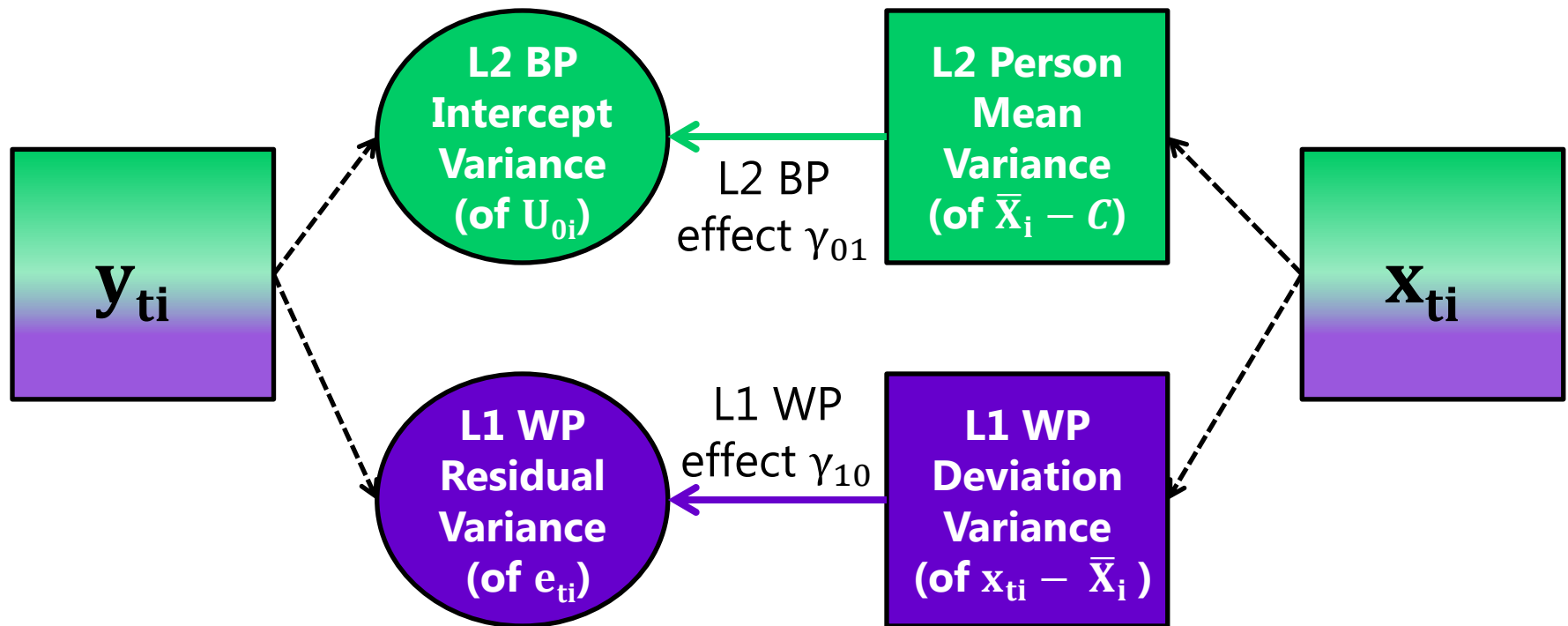
γ_{01} = BP main effect of having more \bar{X}_i than other people

Because WPx_{ti} and PMx_i are uncorrelated, each gets the total effect for its level (WP=L1, BP=L2)

Person-Mean-Centering

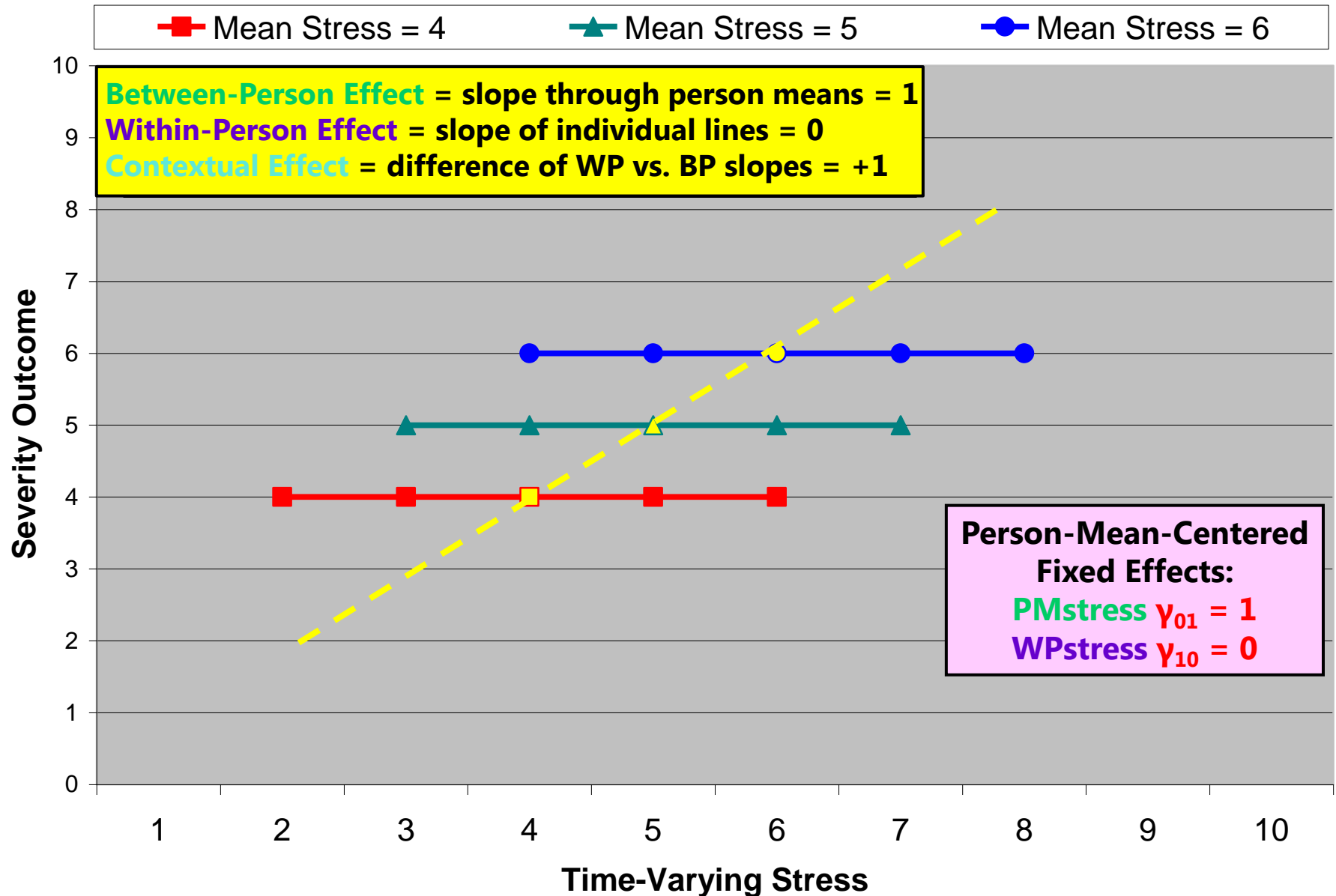
Model-based partitioning of level-1 y_{ti} outcome variance into **variance components**:

Brute-force partitioning of level-1 x_{ti} predictor variance into **observed variables**:

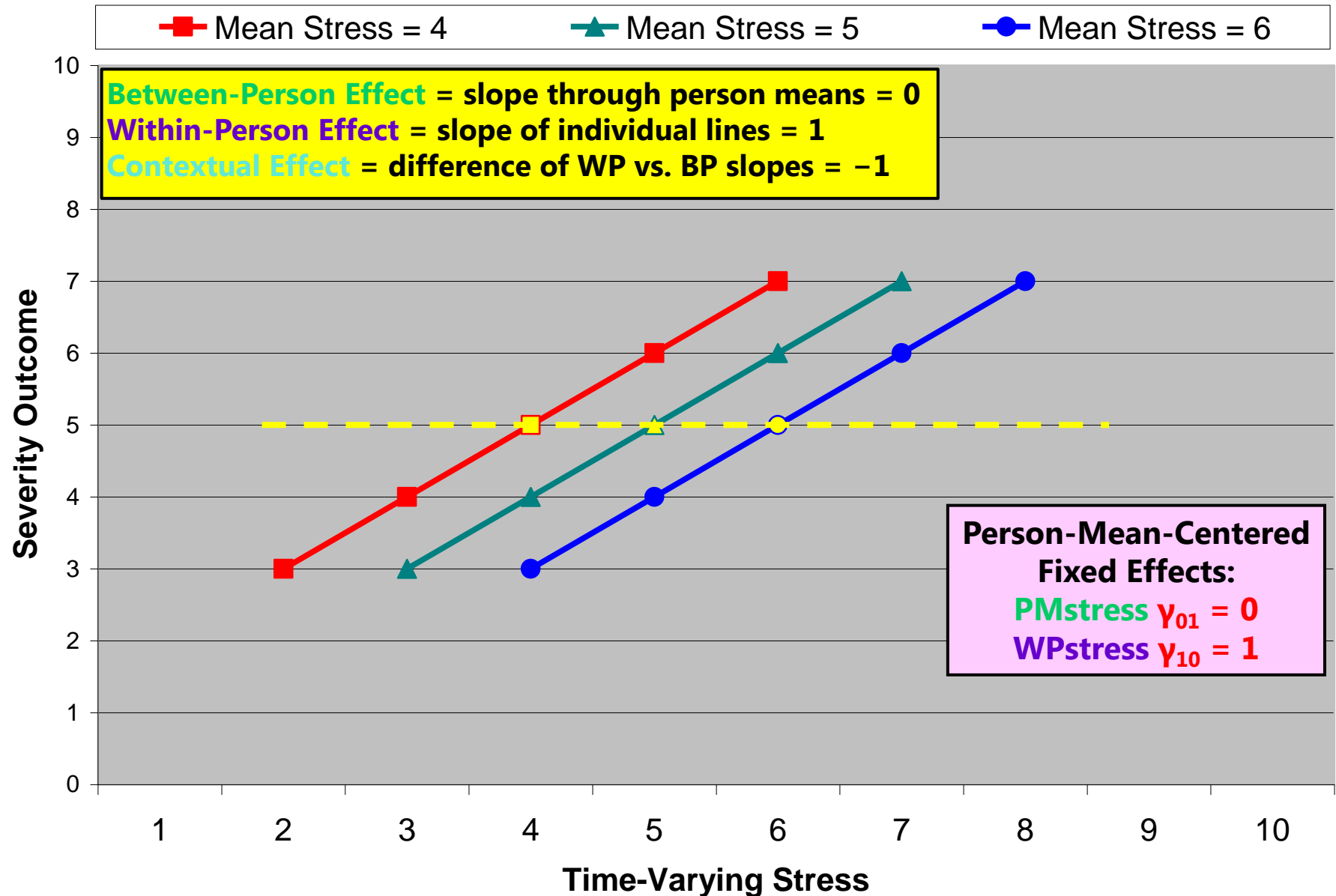


Why not let the model make variance components for x_{ti} , too?
This is the basis of multivariate MLM (or "multilevel SEM"): stay tuned...

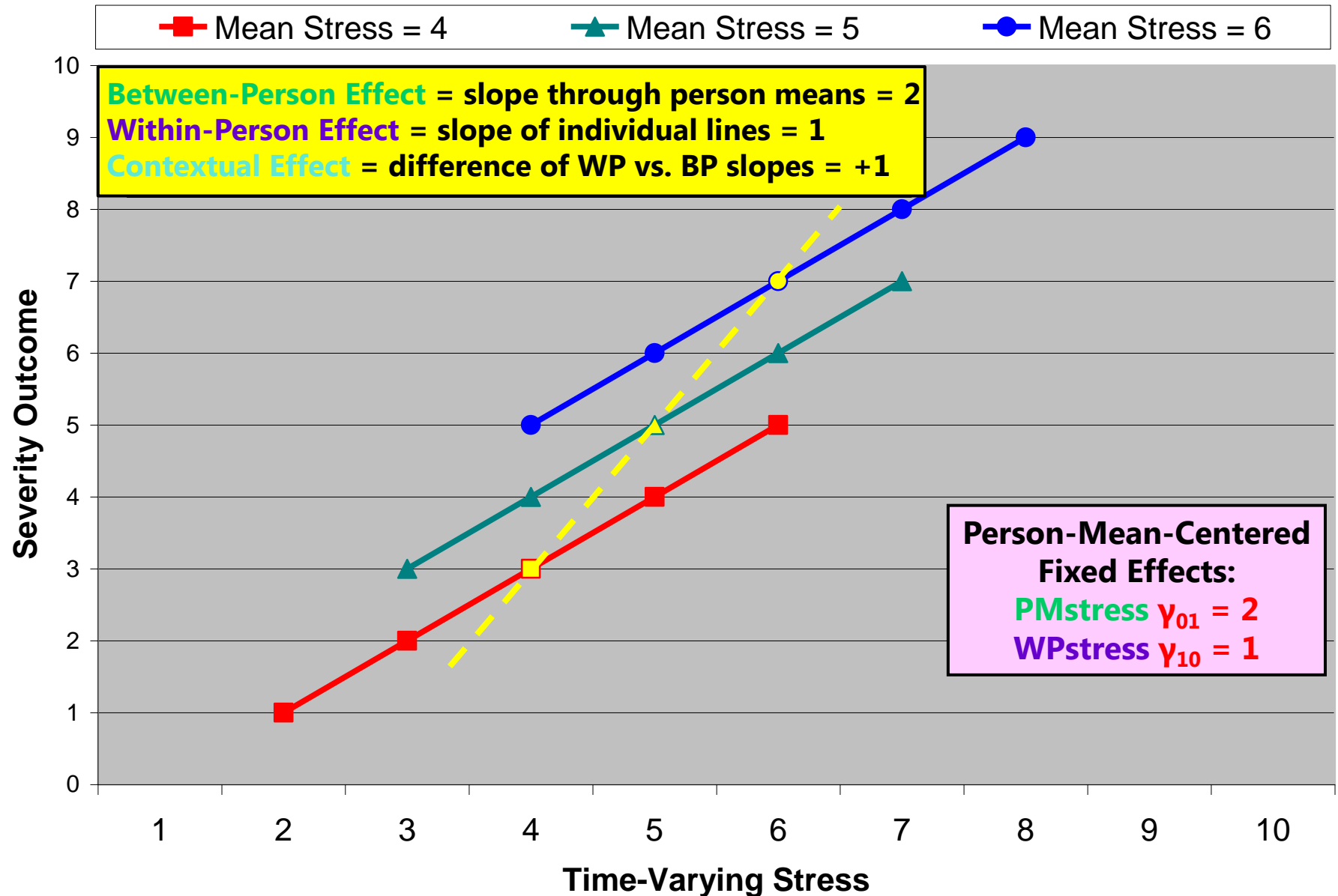
ALL Between-Person Effect, NO Within-Person Effect



NO Between-Person Effect, ALL Within-Person Effect



Between-Person Effect > Within-Person Effect



Within-Person Fluctuation Model with Person-Mean-Centered Level-1 x_{ti}

→ WP and BP Effects directly through separate parameters

x_{ti} is person-mean-centered into WPx_{ti} , with PMx_i at L2:

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}(WPx_{ti}) + e_{ti}$$

$WPx_{ti} = x_{ti} - \bar{X}_i \rightarrow$ it has only Level-1 WP variation

$$\text{Level 2: } \beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + u_{0i}$$

$PMx_i = \bar{X}_i - C \rightarrow$ it has only Level-2 BP variation

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i) + u_{1i}$$

u_{1i} is a random slope for the WP effect of x_{ti}

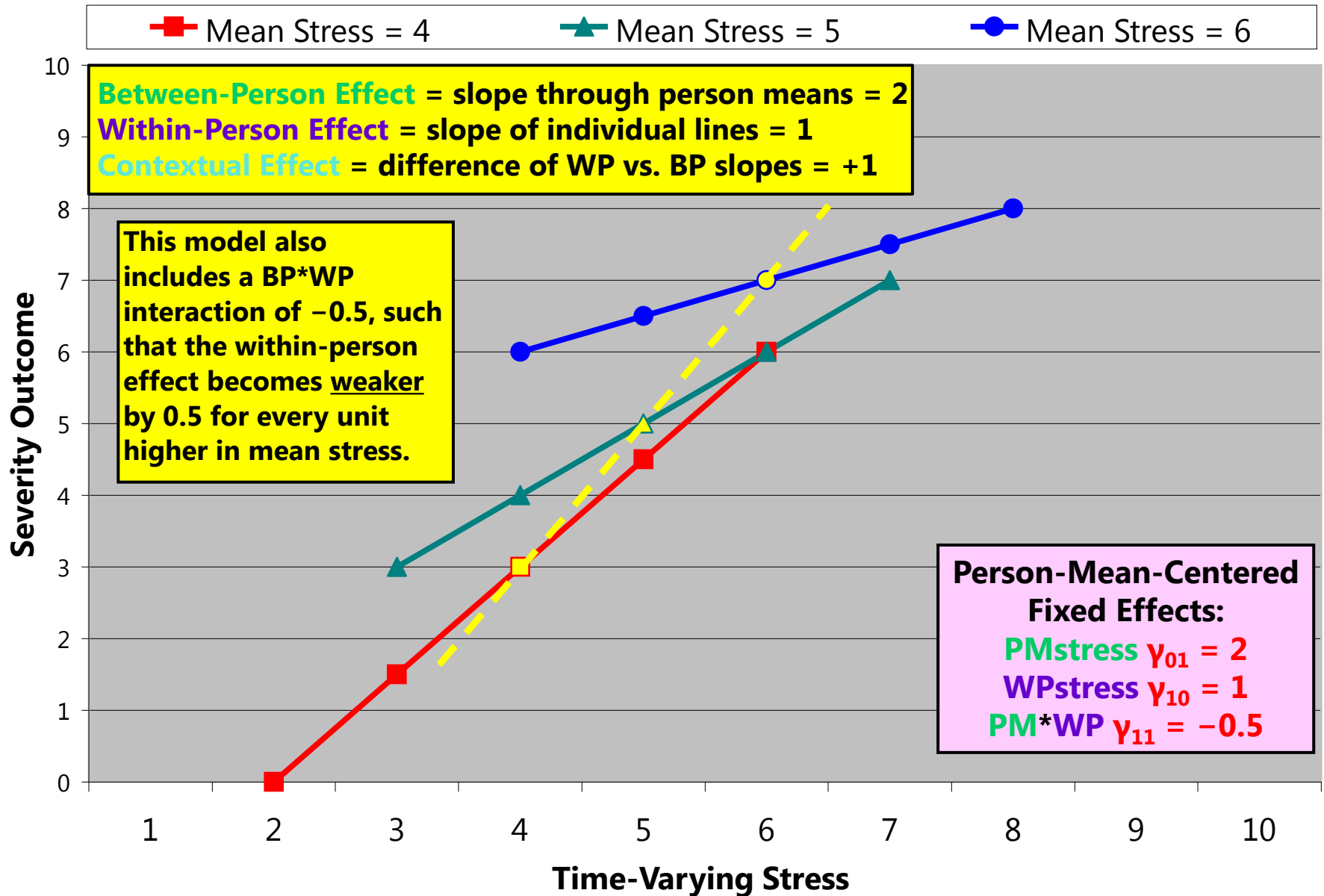
γ_{10} = WP simple main effect of having more x_{ti} than usual for $PMx_i = 0$

γ_{01} = BP simple main effect of having more \bar{X}_i than other people for people at their own mean ($WPx_{ti} = x_{ti} - \bar{X}_i \rightarrow 0$)

γ_{11} = BP*WP interaction: how the effect of having more x_{ti} than usual differs by how much \bar{X}_i you have

Note: this model should also test γ_{02} for $PMx_i * PMx_i$ (stay tuned)

Between-Person x Within-Person Interaction



3 Kinds of Fixed Effects for TV Predictors

- **First 2 effects Person-Mean-Centering tells us directly:**
- **Is the Level-2 Between-Person (BP) effect significant?**
 - Are people with higher predictor values than other people (*on average over time*) also higher on Y than other people (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random intercept variance ($\tau_{U_0}^2$)?
 - This would be indicated by a significant fixed effect of **PM x_i**
 - Note: this is NOT controlling for the absolute value of x_{ti} at each occasion
- **Is the Level-1 Within-Person (WP) effect significant?**
 - If you have higher predictor values than usual (*at this occasion*), do you also have higher outcomes values than usual (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance (σ_e^2)?
 - This would be indicated by a significant fixed effect of **WP x_{ti}**
 - Note: this is represented by the relative value of x_{ti} , NOT the absolute value of x_{ti}

3rd Kind of Effect for TV Predictors

- **What Person-Mean-Centering DOES NOT tell us directly:**
- **Are BP and WP effects different : Is there a level-2 contextual effect?**
 - After controlling for the absolute value of the TV predictor at each occasion, is there still an incremental contribution from having a higher person mean of the TV predictor (i.e., does one's general tendency predict $\tau_{U_0}^2$ above and beyond just the time-specific value of the predictor)?
 - If there is no contextual effect, then the BP and WP effects of the TV predictor show **convergence**, such that their effects are of equivalent magnitude
- **To answer this question about the level-2 contextual effect for the incremental contribution of the person mean, we have two options:**
 - Ask for the contextual effect via an ESTIMATE statement in SAS (or TEST in SPSS, or NEW in Mplus, or LINCOM in STATA): $WP\mathbf{x}_{ti} \quad -1 \quad PM\mathbf{x}_i \quad 1$
 - Use **“grand-mean-centering”** for time-varying \mathbf{x}_{ti} instead: $TV\mathbf{x}_{ti} = \mathbf{x}_{ti} - \mathbf{C}$
→ **centered at a CONSTANT, NOT A LEVEL-2 VARIABLE**
 - Which constant only matters for what the reference point is; it could be the grand mean or other

Why the Difference in the Level-2 Effect?

Remember Regular Old Regression...

- In this model: $Y_i = \beta_0 + \beta_1(X_{1i}) + \beta_2(X_{2i}) + e_i$
- If X_{1i} and X_{2i} **ARE NOT** correlated:
 - β_1 is **ALL the relationship** between X_{1i} and Y_i
 - β_2 is **ALL the relationship** between X_{2i} and Y_i
- If X_{1i} and X_{2i} **ARE** correlated:
 - β_1 is **different than** the full relationship between X_{1i} and Y_i
 - “Unique” effect of X_{1i} *controlling for X_{2i} or holding X_{2i} constant*
 - β_2 is **different than** the full relationship between X_{2i} and Y_i
 - “Unique” effect of X_{2i} *controlling for X_{1i} or holding X_{1i} constant*
- Hang onto that idea...

Person-MC vs. Grand-MC for Time-Varying Predictors

Level 2		Original	Person-MC Level 1	Grand-MC Level 1
\bar{X}_i	$PMx_i = \bar{X}_i - 5$	x_{ti}	$WPx_{ti} = x_{ti} - \bar{X}_i$	$TVx_{ti} = x_{ti} - 5$
3	-2	2	-1	-3
3	-2	4	1	-1
7	2	6	-1	1
7	2	8	1	3

Same PMx_i goes into the model using either way of centering the level-1 variable x_{ti}

Using **Person-MC**, WPx_{ti} has NO level-2 BP variation, so it is not correlated with PMx_i

Using **Grand-MC**, TVx_{ti} STILL has level-2 BP variation, so it is STILL CORRELATED with PMx_i

So the effects of PMx_i and TVx_{ti} when included together under Grand-MC will be different than their effects would be if they were by themselves...

WRONG WAY: Within-Person Fluctuation Model with x_{ti} **represented at Level 1 Only:** → WP and BP Effects are Smushed Together

x_{ti} is grand-mean-centered into TVx_{ti} , WITHOUT PMx_i at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(TVx_{ti}) + e_{ti}$

$TVx_{ti} = x_{ti} - C \rightarrow$ it still has both Level-2 BP and Level-1 WP variation

Level 2: $\beta_{0i} = Y_{00} + U_{0i}$
 $\beta_{1i} = Y_{10} + (U_{1i})$

U_{1i} is a random slope for the smushed effect of x_{ti}

Y_{10} = *smushed* WP and BP effects

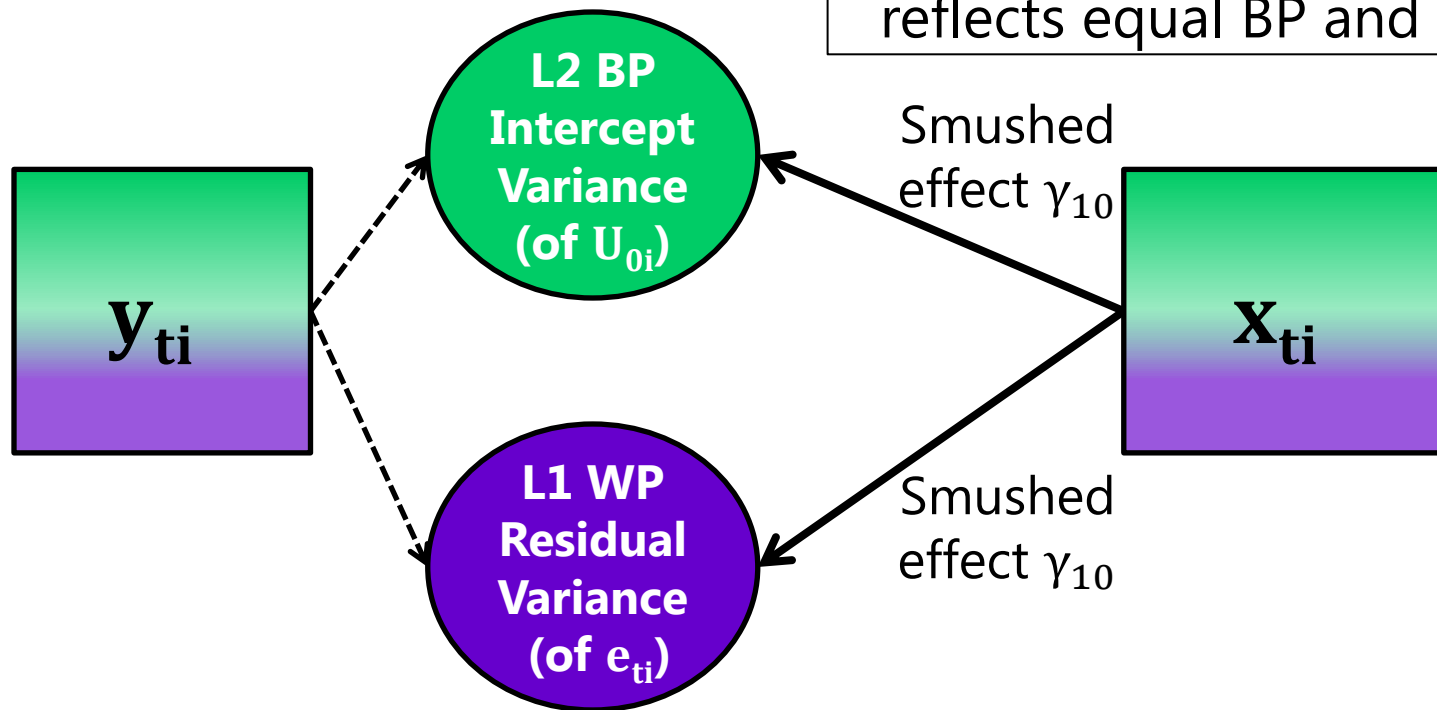
A *smushed* effect is also referred to as the *convergence, conflated, or composite* effect

Because TVx_{ti} still contains its original 2 different kinds of variation (BP and WP), its 1 fixed effect has to do the work of 2 predictors!

Grand-Mean-Centering: Smushing

Model-based partitioning of level-1 y_{ti} outcome variance into **variance components**:

Original level-1 x_{ti} has not been partitioned AND it has only **one fixed effect** coefficient in the model. Thus, that smushed effect reflects equal BP and WP effects.



Convergence (Smushed) Effect of a Time-Varying Predictor

$$\text{Convergence Effect: } \gamma_{\text{conv}} \approx \frac{\frac{\gamma_{\text{BP}}}{\text{SE}_{\text{BP}}^2} + \frac{\gamma_{\text{WP}}}{\text{SE}_{\text{WP}}^2}}{\frac{1}{\text{SE}_{\text{BP}}^2} + \frac{1}{\text{SE}_{\text{WP}}^2}}$$

Adapted from
Raudenbush & Bryk
(2002, p. 138)

- **The convergence effect will often be closer to the within-person effect** (due to larger level-1 sample size and thus smaller SE)
- **It is the rule, not the exception, that between and within effects differ** (Snijders & Bosker, 1999, p. 52-56, and personal experience!)
- However—when grand-mean-centering a time-varying predictor, **convergence is testable** by including a **contextual effect (carried by the person mean)** for how the **BP effect** differs from the **WP effect**...

Within-Person Fluctuation Model with **Grand-Mean-Centered Level-1 x_{ti}**

→ Model tests difference of WVP vs. BP effects (So it's been fixed!)

x_{ti} is grand-mean-centered into TVx_{ti} , WITH PMx_i at L2:

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}(TVx_{ti}) + e_{ti}$$

$TVx_{ti} = x_{ti} - C \rightarrow$ it still has both Level-2 BP and Level-1 WP variation

$$\text{Level 2: } \beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$$

$PMx_i = \bar{x}_i - C \rightarrow$ it has only Level-2 BP variation

$$\beta_{1i} = \gamma_{10} + (U_{1i})$$

U_{1i} is a random slope for the WP effect of x_{ti}

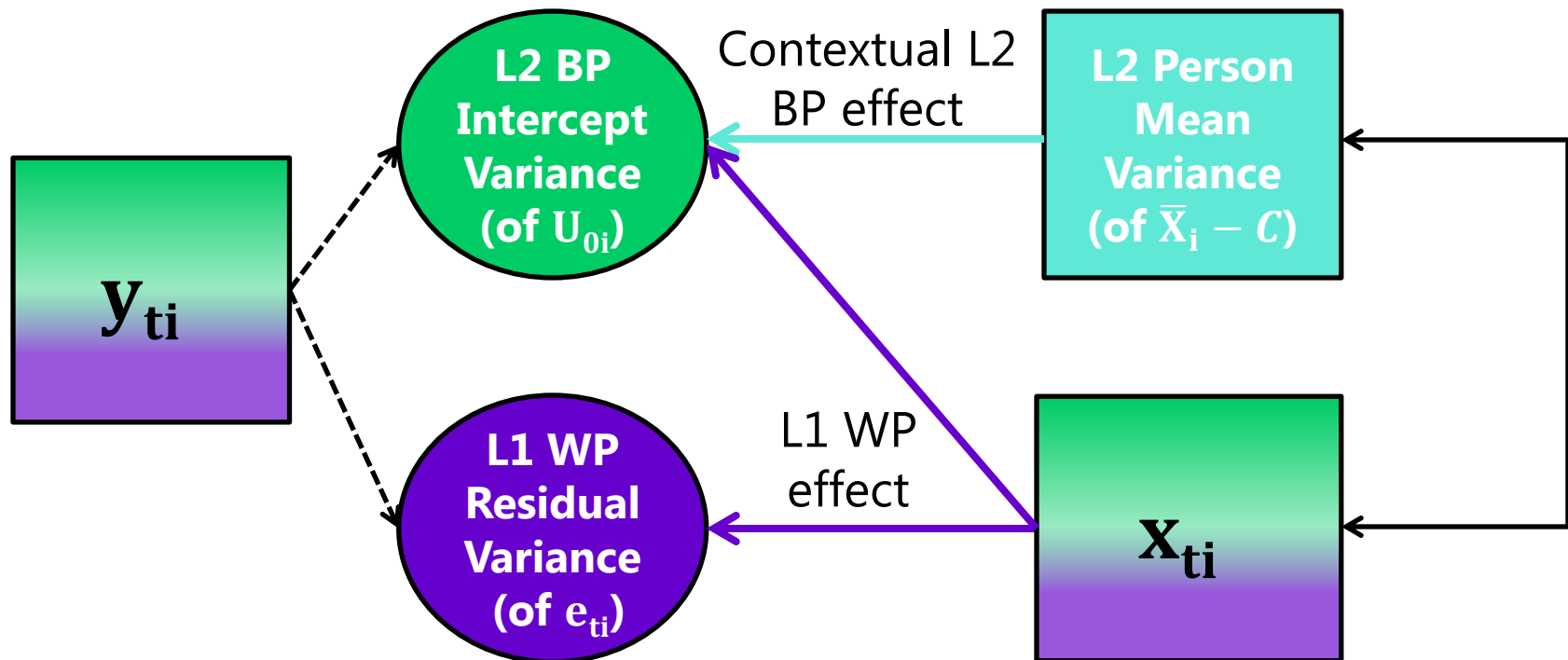
γ_{10} becomes the **WP effect** → *unique* level-1 effect after controlling for PMx_i

γ_{01} becomes the **L2 contextual effect** that indicates how the total BP effect differs from the WP effect → *unique* level-2 effect after controlling for TVx_{ti} → does usual level matter beyond current level?

Grand-Mean-Centering + PM

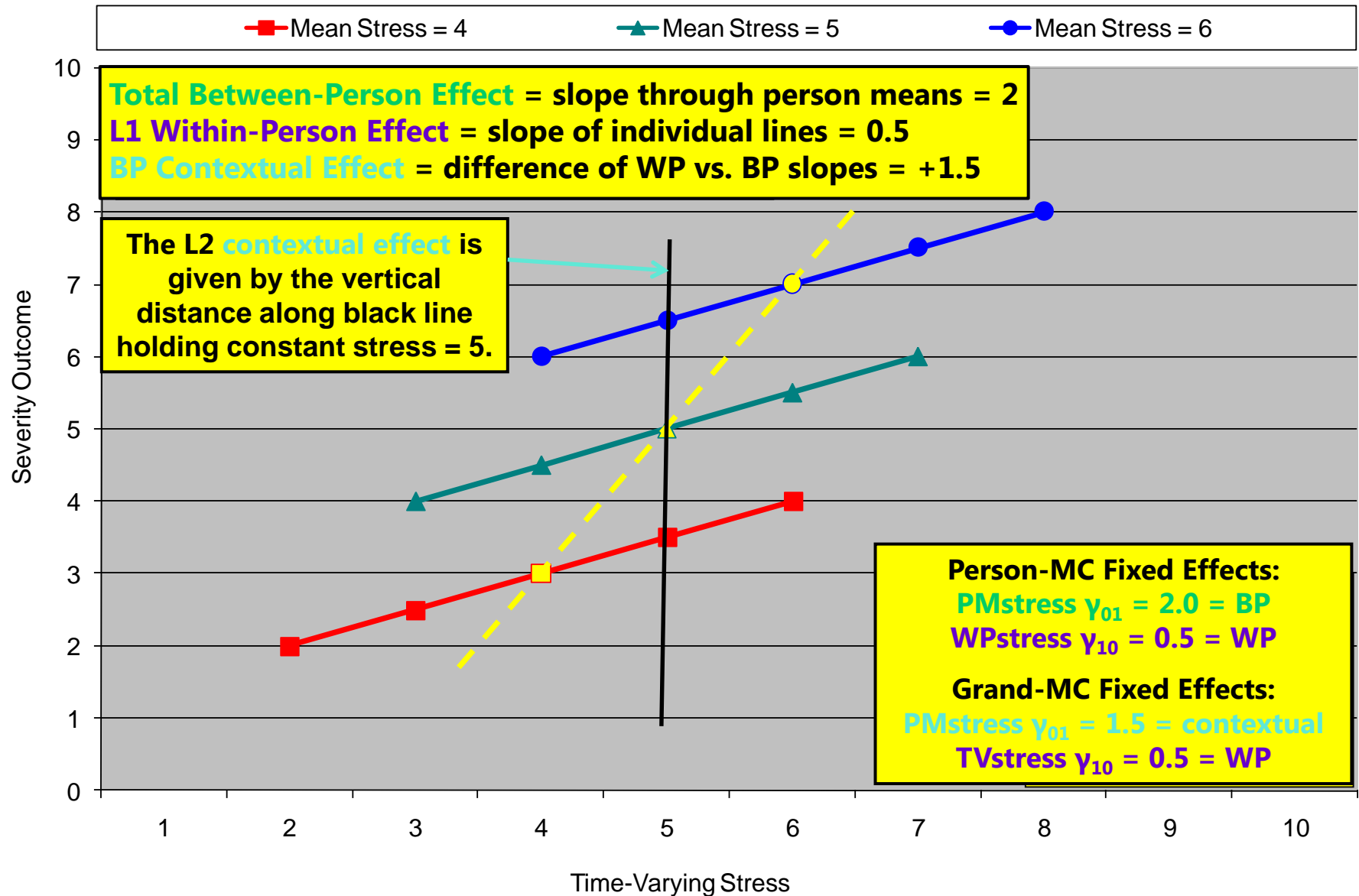
Model-based partitioning of level-1 y_{ti} outcome variance into **variance components**:

Original level-1 x_{ti} is not partitioned, but person mean $\bar{X}_i - C$ is added to allow an extra (different) effect at L2.



Because original x_{ti} still has BP variance, it still carries part of the BP effect...

P-MC vs. G-MC: Interpretation Example



Person-MC and Grand-MC Models are Equivalent Given a **Fixed** Level-1 Main Effect Only

Person-MC: $WP_{x_{ti}} = x_{ti} - PM_{x_i}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PM_{x_i}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PM_{x_i}) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PM_{x_i}) + \gamma_{10}(x_{ti} - PM_{x_i}) + U_{0i} + e_{ti}$

$\rightarrow y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PM_{x_i}) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

Composite Model:

← In terms of P-MC

← In terms of G-MC

Grand-MC: $TV_{x_{ti}} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PM_{x_i}) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PM_{x_i}) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

Effect	P-MC	G-MC
Intercept	γ_{00}	γ_{00}
WP Effect	γ_{10}	γ_{10}
Contextual	$\gamma_{01} - \gamma_{10}$	γ_{01}
BP Effect	γ_{01}	$\gamma_{01} + \gamma_{10}$

Summary: 3 Fixed Effects for TV Predictors

- **Is the Between-Person (BP; Level 2) effect significant?**

- Are people with higher predictor values than other people (*on average over time*) also higher on Y than other people (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random intercept variance ($\tau_{U_0}^2$)?
- Given directly by level-2 effect of PM_{x_i} if using Person-MC for the level-1 predictor (or can be requested via ESTIMATE if using Grand-MC for the level-1 predictor)

- **Is the Within-Person (WP; Level 1) effect significant?**

- If you have higher predictor values than usual (*at this occasion*), do you also have higher outcomes values than usual (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance (σ_e^2)?
- Given directly by the level-1 effect of $WP_{x_{ti}}$ if using Person-MC —OR— given directly by the level-1 effect of $TV_{x_{ti}}$ if using Grand-MC and including PM_{x_i} at level 2 (without PM_{x_i} , the level-1 effect of $TV_{x_{ti}}$ if using Grand-MC is the smushed effect)

- **Are the BP and WP Effects different: Is there a level-2 contextual effect?**

- After controlling for the absolute value of TV predictor value at each occasion, is there still an incremental contribution from having a higher person mean of the TV predictor (i.e., does one's general tendency predict $\tau_{U_0}^2$ above and beyond)?
- Given directly by level-2 effect of PM_{x_i} if using Grand-MC for the level-1 predictor (or can be requested via ESTIMATE if using Person-MC for the level-1 predictor)

Variance Accounted For By Level-2 Predictors

- **Total- R^2 can provide an across-level effect size**
 - = correlation² of original outcome and outcome predicted from the model fixed effects)
- **Fixed effects of level 2 predictors *by themselves*:**
 - Level-2 (BP) main effects reduce level-2 (BP) random intercept variance
 - Level-2 (BP) interactions also reduce level-2 (BP) random intercept variance
- **Fixed effects of *cross-level interactions* (level 1* level 2):**
 - Always test the level-2 random slope variance for the level-1 predictor first!
 - If the interacting level-1 predictor is random, any cross-level interaction with it will reduce its corresponding level-2 BP random slope variance (that line's U)
 - If the interacting level-1 predictor not random, any cross-level interaction with it will reduce the level-1 WP residual variance instead
 - The level-1 effect would then be called "**systematically varying**" to reflect a compromise between "fixed" (all the same) and "random" (all different)—it's not that each person needs his or her own slope, but that the slope varies systematically across people as a function of a known person predictor (and not otherwise)

Variance Accounted For By Level-1 Predictors

- **Fixed effects of level 1 predictors *by themselves*:**
 - Level-1 (WP) main effects reduce Level-1 (WP) residual variance
 - Level-1 (WP) interactions also reduce Level-1 (WP) residual variance
- **What happens at level 2 depends on what levels of variance the level-1 predictor has:**
 - If the level-1 predictor ALSO has level-2 variance (e.g., Grand-MC predictors), then its level-2 variance will also likely reduce level-2 random intercept variance
 - If the level-1 predictor DOES NOT have level-2 variance (e.g., Person-MC predictors), then its reduction in the level-1 residual variance will cause an INCREASE in level-2 random intercept variance
 - Same thing happens with Grand-MC level-1 predictors, but you don't generally see it
 - It's just an artifact that the estimate of true random intercept variance is:
$$\text{True } \tau_{U_0}^2 = \text{observed } \tau_{U_0}^2 - \frac{\sigma_e^2}{n} \quad \rightarrow \text{ so if only } \sigma_e^2 \text{ decreases, } \tau_{U_0}^2 \text{ increases}$$

The Joy of Interactions Involving Time-Varying Predictors

- **Must consider interactions with both its BP and WP parts:**
- Example: Does time-varying stress (x_{ti}) interact with sex (Sex_i)?
- Person-Mean-Centering:
 - $WPx_{ti} * Sex_i \rightarrow$ Does the WP stress effect differ between men and women?
 - $PMx_i * Sex_i \rightarrow$ Does the BP stress effect differ between men and women?
 - Not controlling for current levels of stress
 - If forgotten, then Sex_i moderates the stress effect only at level 1 (WP, not BP)
- Grand-Mean-Centering:
 - $TVx_{ti} * Sex_i \rightarrow$ Does the WP stress effect differ between men and women?
 - $PMx_i * Sex_i \rightarrow$ Does the *contextual* stress effect differ b/t men and women?
 - Incremental BP stress effect *after controlling for current levels of stress*
 - If forgotten, then although the level-1 main effect of stress has been un-smushed via the main effect of PMx_i , the interaction of $TVx_{ti} * Sex_i$ would still be smushed

Interactions with Time-Varying Predictors: Example: TV Stress (x_{ti}) by Gender (Sex_i)

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + U_{0i}$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(Sex_i)$$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
 $+ \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti} - PMx_i)$

Grand-MC: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + U_{0i}$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(Sex_i)$$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
 $+ \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti})$

Interactions Involving Time-Varying Predictors Belong at Both Levels of the Model

On the left below → Person-MC: $WP_{x_{ti}} = x_{ti} - PM_{x_i}$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PM_{x_i}) + \gamma_{10}(x_{ti} - PM_{x_i}) + U_{0i} + e_{ti} \\ + \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PM_{x_i}) + \gamma_{11}(Sex_i)(x_{ti} - PM_{x_i})$$

← Composite model
written as Person-MC

$$y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PM_{x_i}) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti} \\ + \gamma_{02}(Sex_i) + (\gamma_{03} - \gamma_{11})(Sex_i)(PM_{x_i}) + \gamma_{11}(Sex_i)(x_{ti})$$

← Composite model
written as Grand-MC

On the right below → Grand-MC: $TV_{x_{ti}} = x_{ti}$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PM_{x_i}) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti} \\ + \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PM_{x_i}) + \gamma_{11}(Sex_i)(x_{ti})$$

After adding an interaction for Sex_i with stress at both levels, then the Person-MC and Grand-MC models are equivalent

Intercept: $\gamma_{00} = \gamma_{00}$

BP Effect: $\gamma_{01} = \gamma_{01} + \gamma_{10}$

Contextual: $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WP Effect: $\gamma_{10} = \gamma_{10}$ BP*Sex Effect: $\gamma_{03} = \gamma_{03} + \gamma_{11}$ Contextual*Sex: $\gamma_{03} = \gamma_{03} - \gamma_{11}$

Sex Effect: $\gamma_{20} = \gamma_{20}$

BP*WP or Contextual*WP is the same: $\gamma_{11} = \gamma_{11}$

Intra-variable Interactions

- **Still must consider interactions with both its BP and WP parts!**
- Example: Interaction of TV stress (x_{ti}) with person mean stress (PMx_i)
- Person-Mean-Centering:
 - $WPx_{ti} * PMx_i \rightarrow$ Does the WP stress effect differ by overall stress level?
 - $PMx_i * PMx_i \rightarrow$ Does the BP stress effect differ by overall stress level?
 - Not controlling for current levels of stress
 - If forgotten, then PMx_i moderates the stress effect only at level 1 (WP, not BP)
- Grand-Mean-Centering:
 - $TVx_{ti} * PMx_i \rightarrow$ Does the WP stress effect differ by overall stress level?
 - $PMx_i * PMx_i \rightarrow$ Does the *contextual* stress effect differ by overall stress?
 - Incremental BP stress effect *after controlling for current levels of stress*
 - If forgotten, then although the level-1 main effect of stress has been un-smushed via the main effect of PMx_i , the interaction of $TVx_{ti} * PMx_i$ would still be smushed

Intra-variable Interactions:

Example: TV Stress (x_{ti}) by Person Mean Stress (PMx_i)

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(PMx_i)(PMx_i) + U_{0i}$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i)$$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
 $+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti} - PMx_i)$

Grand-MC: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(PMx_i)(PMx_i) + U_{0i}$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i)$$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
 $+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$

Intra-variable Interactions:

Example: TV Stress (x_{ti}) by Person Mean Stress (PMx_i)

On the left below → Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti} \\ + \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti} - PMx_i)$$

← Written as
Person-MC

$$y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti} \\ + (\gamma_{02} - \gamma_{11})(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$$

← Written as
Grand-MC

On the right below → Grand-MC: $TVx_{ti} = x_{ti}$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti} \\ + \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$$

After adding an interaction for PMx_i with stress at both levels, then the Person-MC and Grand-MC models are equivalent

Intercept: $\gamma_{00} = \gamma_{00}$

BP Effect: $\gamma_{01} = \gamma_{01} + \gamma_{10}$

Contextual: $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WP Effect: $\gamma_{10} = \gamma_{10}$

BP² Effect: $\gamma_{02} = \gamma_{02} + \gamma_{11}$

Contextual²: $\gamma_{02} = \gamma_{02} - \gamma_{11}$

BP*WP or Contextual*WP is the same: $\gamma_{11} = \gamma_{11}$

When Person-MC \neq Grand-MC: Random Effects of TV Predictors

Person-MC: $WP_{\mathbf{x}_{ti}} = \mathbf{x}_{ti} - PM_{\mathbf{x}_i}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{x}_{ti} - PM_{\mathbf{x}_i}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PM_{\mathbf{x}_i}) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

Variance due to $PM_{\mathbf{x}_i}$ is removed from the random slope in Person-MC.

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PM_{\mathbf{x}_i}) + \gamma_{10}(\mathbf{x}_{ti} - PM_{\mathbf{x}_i}) + U_{0i} + U_{1i}(\mathbf{x}_{ti} - PM_{\mathbf{x}_i}) + e_{ti}$

Grand-MC: $TV_{\mathbf{x}_{ti}} = \mathbf{x}_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{x}_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PM_{\mathbf{x}_i}) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

Variance due to $PM_{\mathbf{x}_i}$ is still part of the random slope in Grand-MC. So these models cannot be made equivalent.

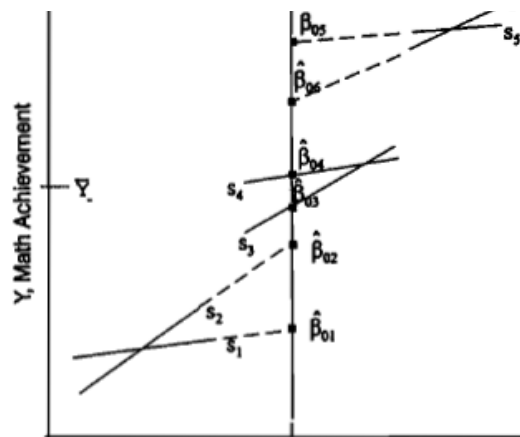
$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PM_{\mathbf{x}_i}) + \gamma_{10}(\mathbf{x}_{ti}) + U_{0i} + U_{1i}(\mathbf{x}_{ti}) + e_{ti}$

Random Effects of TV Predictors

- **Random intercepts** mean different things under each model:
 - **Person-MC** → Individual differences at $\mathbf{WP}\mathbf{x}_{ti} = \mathbf{0}$ (that everyone has)
 - **Grand-MC** → Individual differences at $\mathbf{TV}\mathbf{x}_{ti} = \mathbf{0}$ (that not everyone has)
- **Differential shrinkage of the random intercepts** results from differential reliability of the intercept data across models:
 - Person-MC → Won't affect shrinkage of slopes unless highly correlated
 - Grand-MC → Will affect shrinkage of slopes due to forced extrapolation
- As a result, the **random slope variance may be too small** when using Grand-MC rather than Person-MC
 - Problem worsens with greater ICC of TV Predictor (more extrapolation)
 - Anecdotal example using clustered data was presented in Raudenbush & Bryk (2002; chapter 6)

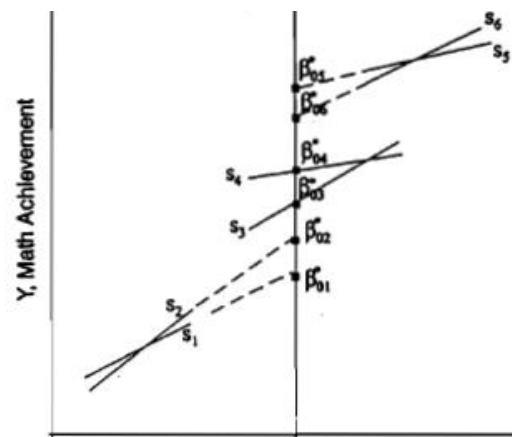
Bias in Random Slope Variance

OLS Per-Person Estimates



Level-1 X

EB Shrunk Estimates



Level-1 X

Top right: Intercepts and slopes are homogenized in Grand-MC because of intercept extrapolation

Bottom: Downwardly-biased random slope variance in Grand-MC relative to Person-MC

Unconditional Results

Conditional Results

Person-MC

$$\hat{\mathbf{T}} = \begin{bmatrix} 8.68 & 0.05 \\ 0.05 & 0.68 \end{bmatrix}$$

$$\hat{\sigma}^2 = 36.70$$

$$\hat{\mathbf{T}} = \begin{bmatrix} 2.38 & 0.19 \\ 0.19 & 0.15 \end{bmatrix}$$

$$\hat{\sigma}^2 = 36.70$$

Grand-MC

$$\hat{\mathbf{T}} = \begin{bmatrix} 4.83 & -0.15 \\ -0.15 & 0.42 \end{bmatrix}$$

$$\hat{\sigma}^2 = 36.83$$

$$\hat{\mathbf{T}} = \begin{bmatrix} 2.41 & 0.19 \\ 0.19 & 0.06 \end{bmatrix}$$

$$\hat{\sigma}^2 = 36.74$$

Modeling Time-Varying Categorical Predictors

- Person-MC and Grand-MC really only apply to *continuous* TV predictors, but the need to consider BP and WP effects applies to *categorical* TV predictors too
- Binary level-1 predictors do not lend themselves to Person-MC
 - e.g., $x_{ti} = 0$ or 1 per occasion, person mean = $.50$ across occasions \rightarrow impossible values
 - If $x_{ti} = 0$, then $WPx_{ti} = 0 - .50 = -0.50$; If $x_{ti} = 1$, then $WPx_{ti} = 1 - .50 = 0.50$
 - Better: Leave x_{ti} uncentered and include person mean as level-2 predictor (results \sim Grand-MC)
- For >2 categories, person means of multiple dummy codes starts to break down, but we can think about types of people, and code BP effects accordingly
- Example: Dementia present/not at each time point?
 - **BP effects** \rightarrow Ever diagnosed with dementia (no, yes)?
 - People who will eventually be diagnosed may differ prior to diagnosis (a BP effect)
 - **TV effect** \rightarrow Diagnosed with dementia at each time point (no, yes)?
 - Acute differences of before/after diagnosis logically can only exist in the “ever” people
- Other examples: Mentor status, father absence, type of shift work (AM/PM)

Wrapping Up: Person-MC vs. Grand-MC

- Time-varying predictors carry at least two potential effects:
 - Some people are higher/lower than other people → BP, level-2 effect
 - Some occasions are higher/lower than usual → WP, level-1 effect
- BP and WP effects almost always need to be represented by two or more model parameters, using either:
 - *Person-mean-centering* ($WP_{x_{ti}}$ and PM_{x_i}): $WP \neq 0?$, $BP \neq 0?$
 - *Grand-mean-centering* ($TV_{x_{ti}}$ and PM_{x_i}): $WP \neq 0?$, $BP \neq WP?$
 - Both yield equivalent models if the level-1 WP effect is fixed, but not if the level-1 WP effect is random
 - Grand MC → *absolute* effect of x_{ti} varies randomly over people
 - Person MC → *relative* effect of x_{ti} varies randomly over people
 - Use prior theory and empirical data (ML AIC, BIC) to decide