Measuring Individual Change: A Gentle Introduction to the Pros and Cons of Modern Models

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Overview

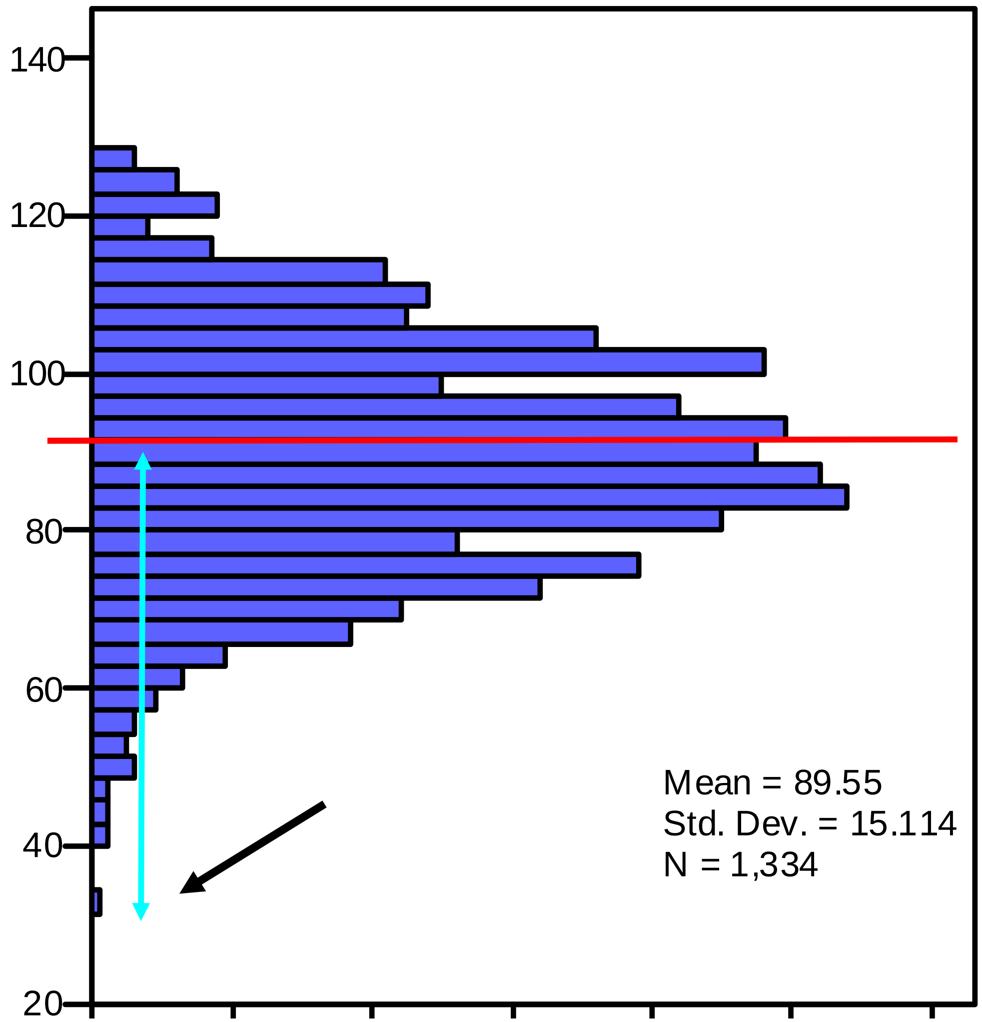
- Organizing principles
  - From one to many kinds of variance
- From cross-sectional to longitudinal (growth) models
  - Multilevel/hierarchical linear/latent growth curve models
- Models for exploring "heterogeneous populations"
  - Latent class/profile/transition/mixture models
- Models for confirming/testing heterogeneous groups
  - Heterogeneous variance longitudinal models
  - Confirmatory longitudinal mixture models

The Two Sides of Any Model

- **Model for the Means:**
  - *Aka* **Fixed Effects**, Structural Part of Model
  - What you are used to **caring about for testing hypotheses**
  - How the expected outcome for a given observation varies as a function of values for **known** predictor variables

- **Model for the Variance:**
  - *Aka* **Random Effects and Residuals**, Stochastic Part of Model
  - What you are used to **making assumptions about** instead
  - How residuals are distributed and related across observations (groups, persons, time, etc.) → these relationships are called "dependency" and ***this is the primary way that longitudinal models differ from general linear models*** *(e.g., regression)*

An Empty Between-Person Model (i.e., Single-Level)



$$y_i = \beta_0 + e_i$$

Filling in values:

$$32 = \underbrace{90}_{y_{pred}} + -58$$

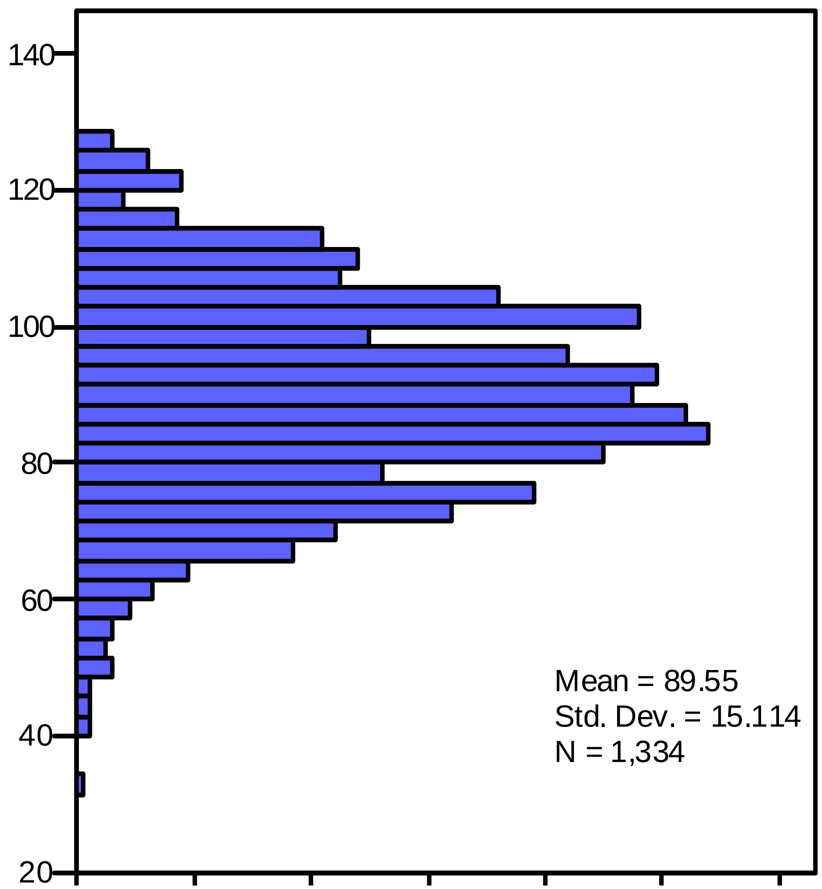
Model for the Means

y_i error variance:

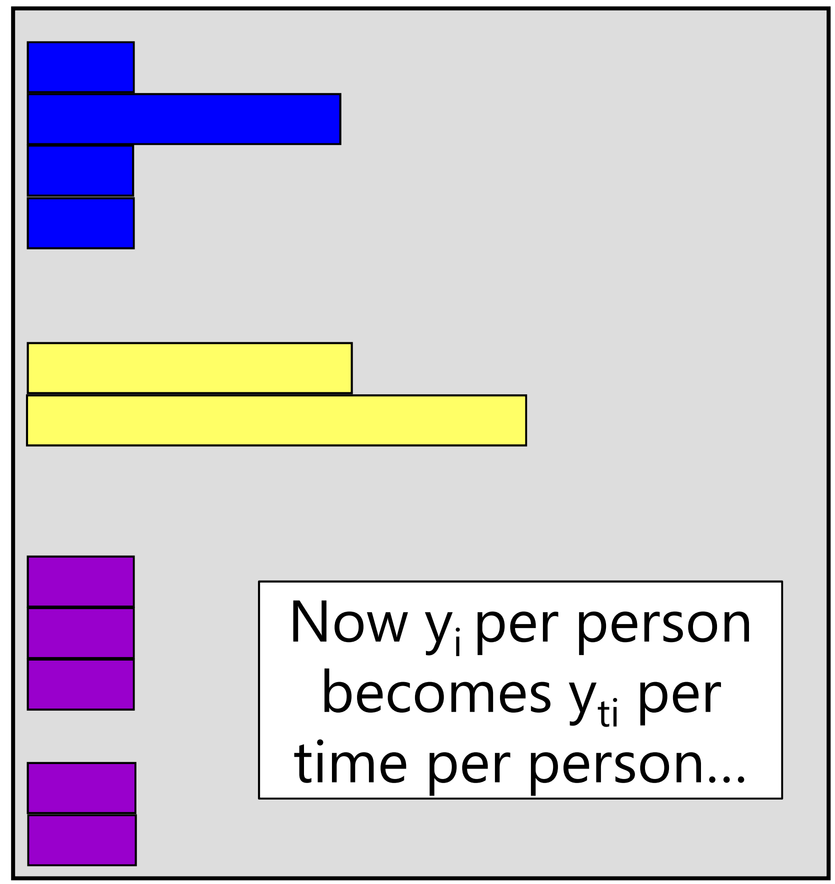
$$\frac{\sum\left(y_i - y_{pred}\right)^2}{N - 1}$$

Adding Within-Person Information… (i.e., to become a Multilevel Model)

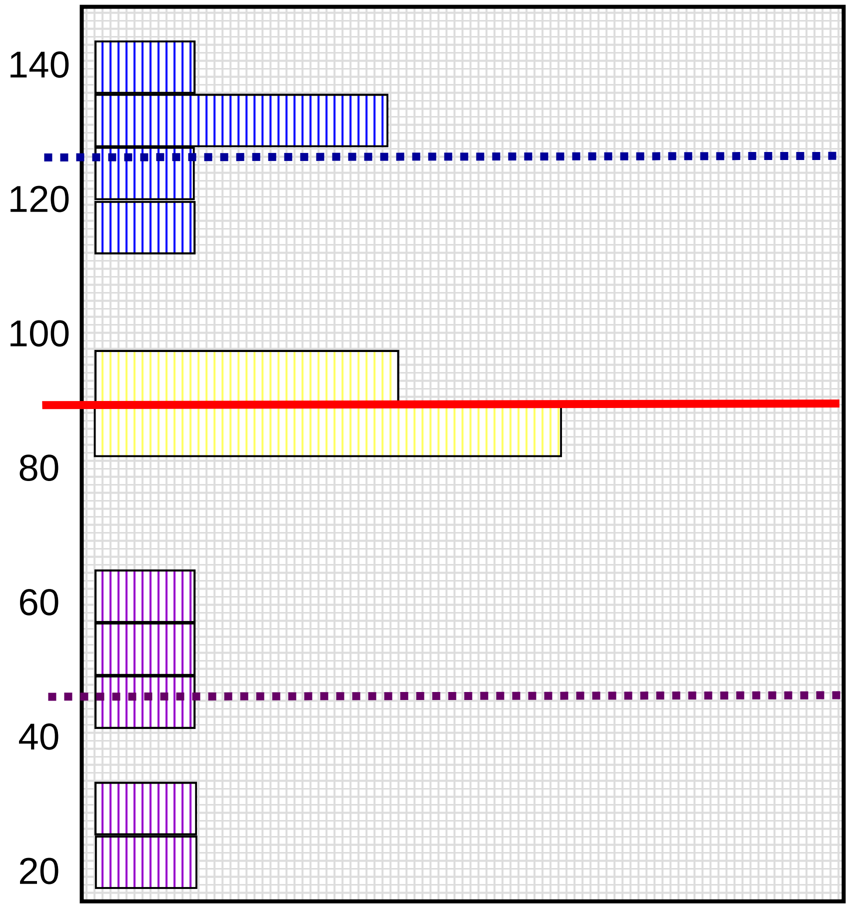
Full Sample Distribution



3 People, 5 Occasions each



Empty +Within-Person Model for y_{ti}



Start off with mean of y_{ti} as "best guess" for any value:

= Grand Mean

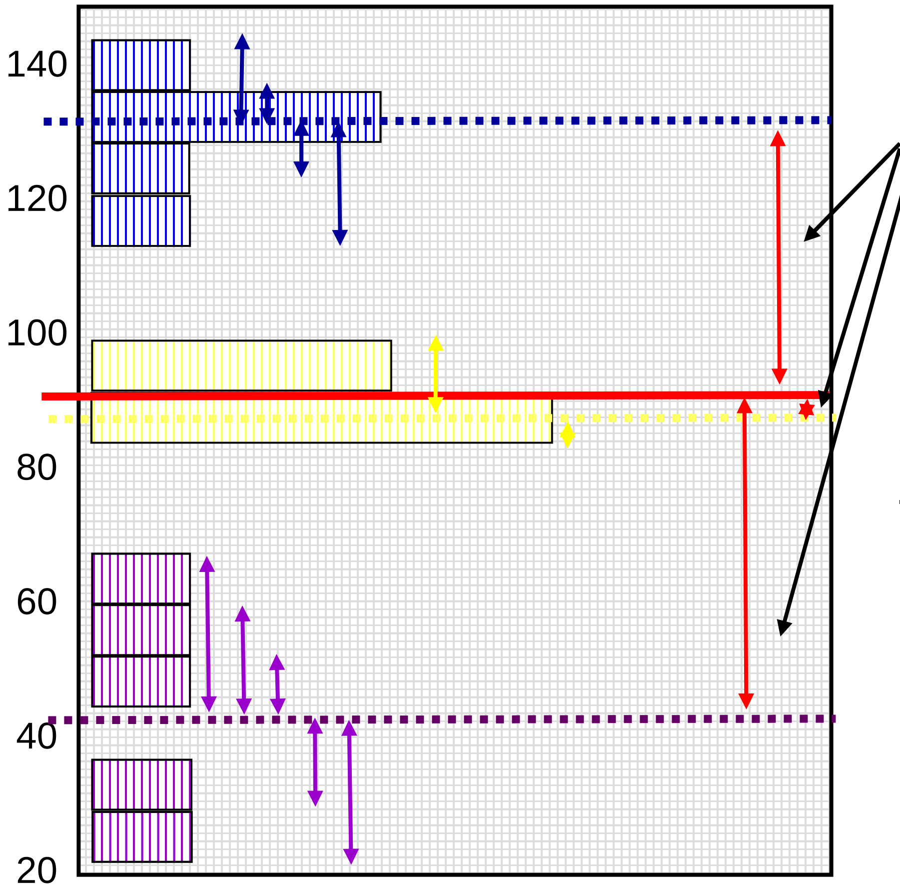
= Fixed Intercept

Can make better guess by taking advantage of repeated observations:

= Person Mean

→ Random Intercept

Empty +Within-Person Model



y_{ti} variance → 2 sources:

Between-Person (BP) Variance:

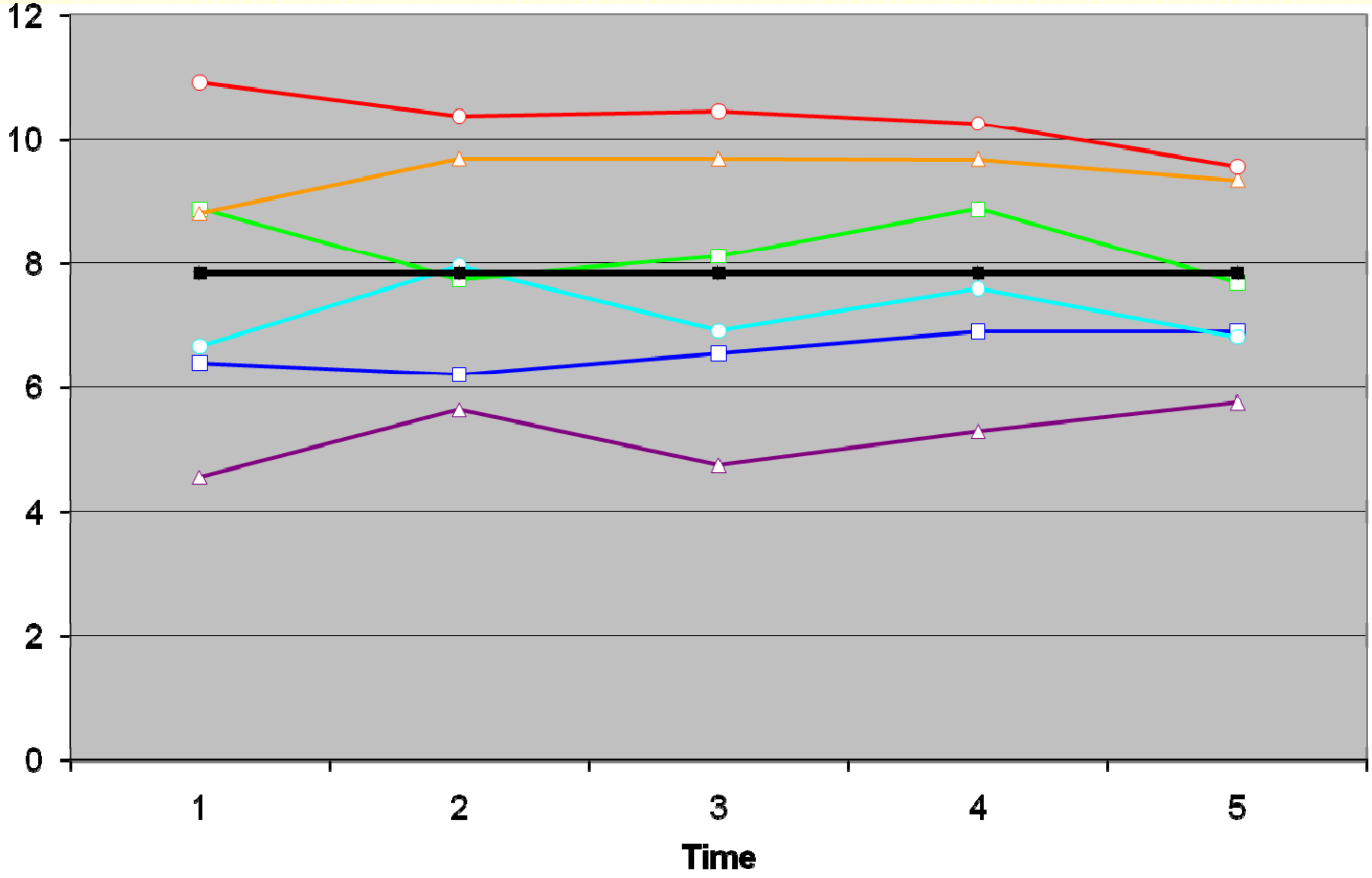
- → Differences from **GRAND** mean
- → **INTER**-Individual Differences

Within-Person (WP) Variance:

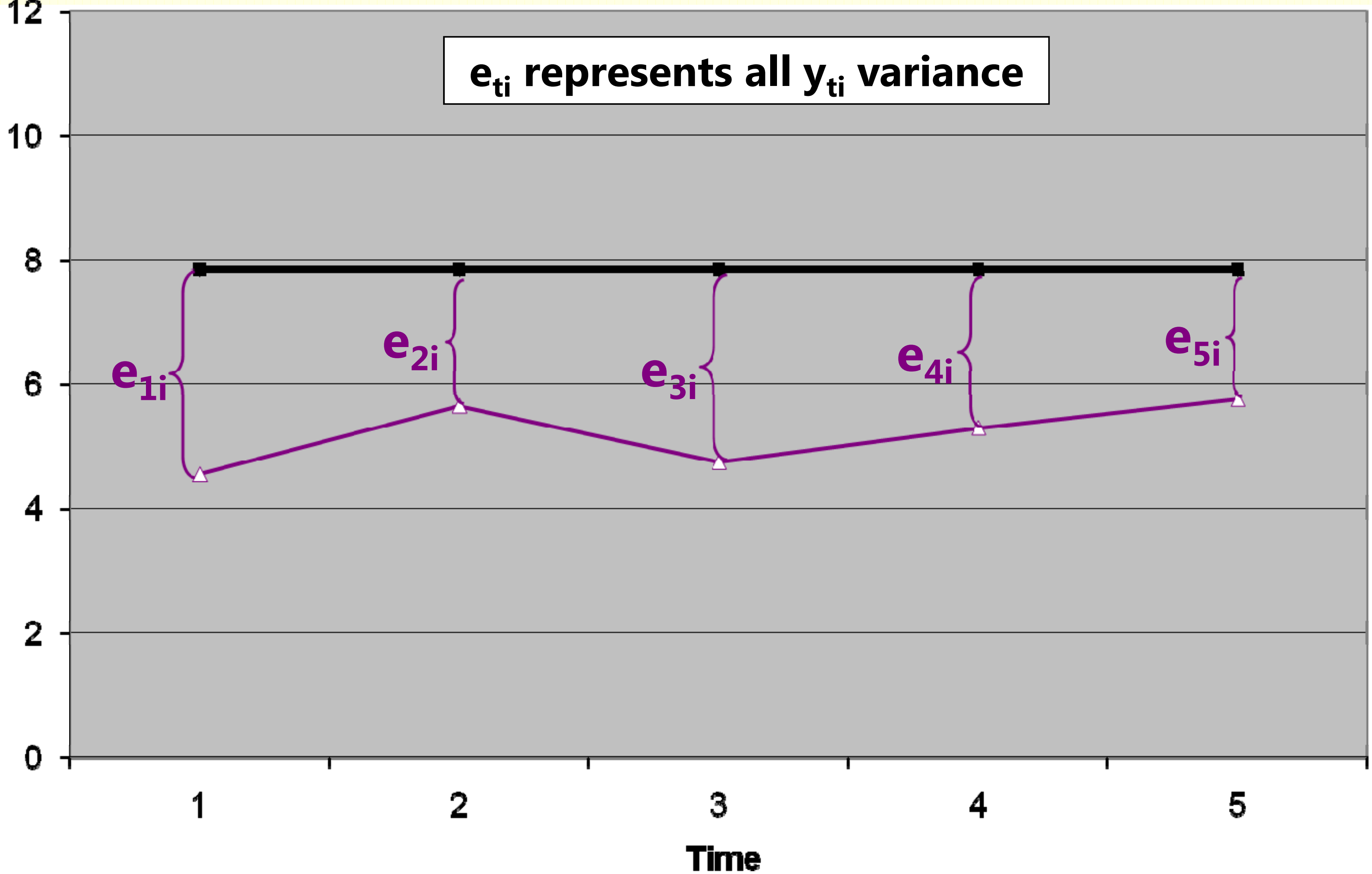
- → Differences from **OWN** mean
- → **INTRA**-Individual Differences
- → This part is only observable through longitudinal data.

Now we have 2 piles of variance in y_{ti} to predict.

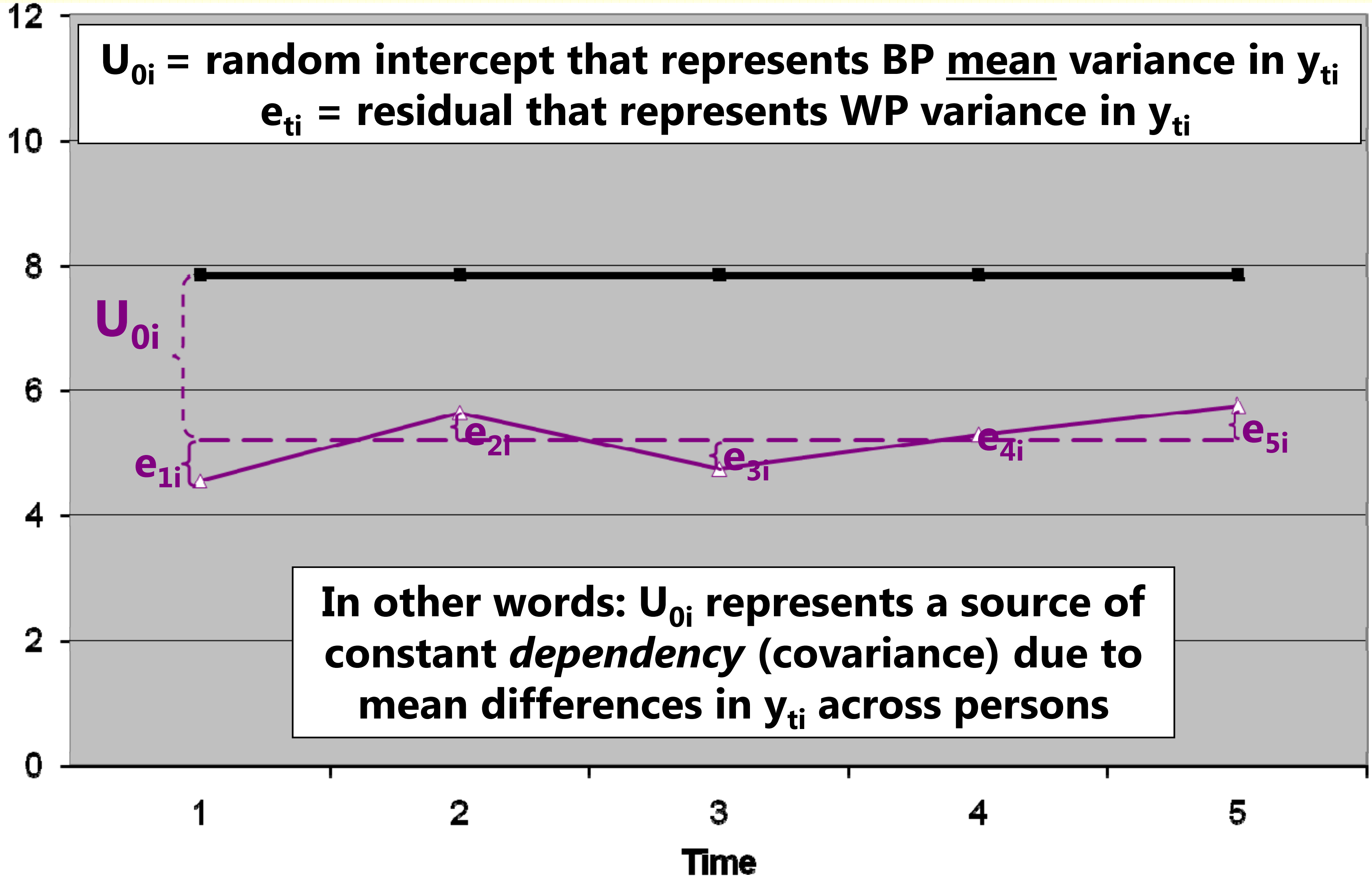
Hypothetical Longitudinal Data (black line = sample mean)



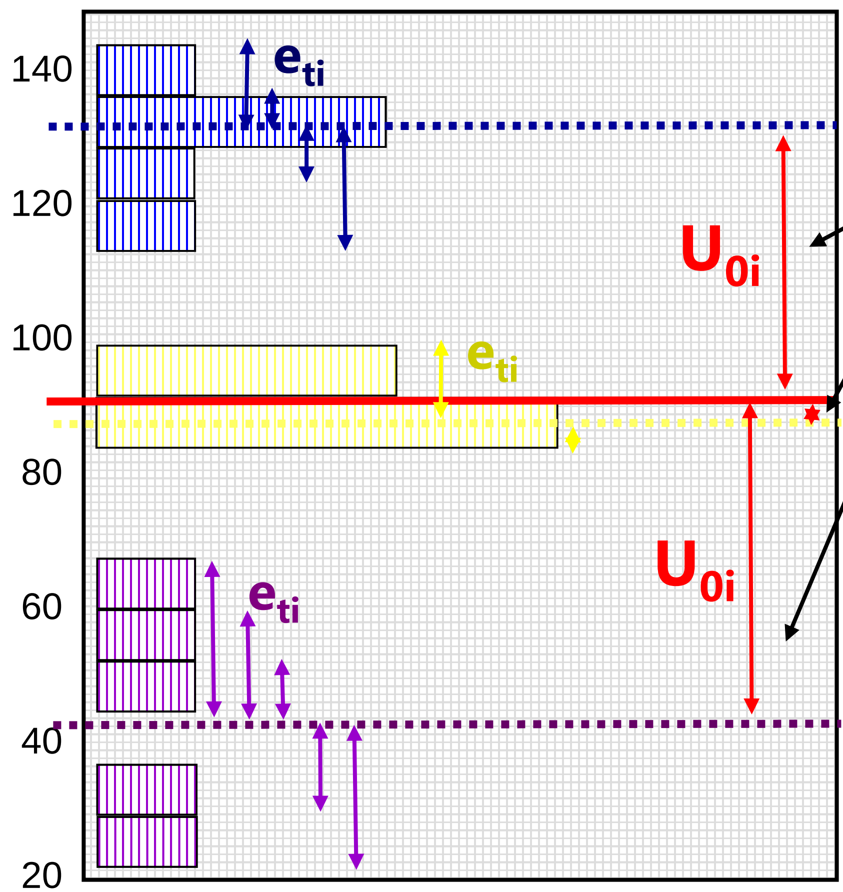
"Error" in a BP Model for the Variance: Single-Level Model



“Error” in a +WP Model for the Variance: Multilevel Model



Empty +Within-Person Model



y_{ti} variance → 2 sources:

Level 2 Random Intercept Variance (of U_{0i}, as $\tau^2_{U_0}$):

→ **Between**-Person Variance

→ Differences from **GRAND** mean

→ **INTER**-Individual Differences

Level 1 Residual Variance (of e_{ti}, as σ^2_e):

→ **Within**-Person Variance

→ Differences from **OWN** mean

→ **INTRA**-Individual Differences

BP and +WP Conditional Models

- Multiple Regression, **Between-Person** ANOVA: **1 PILE**
  - $y_i = (\beta_0 + \beta_1 X_i + \beta_2 Z_i \ldots) + e_i$
  - e_i → ONE residual, assumed uncorrelated with equal variance across observations (here, just persons) → "**BP (all) variation**"

- Repeated Measures, **Within-Person** ANOVA: **2 PILES**
  - $y_{ti} = (\beta_0 + \beta_1 X_i + \beta_2 Z_i \ldots) + U_{0i} + e_{ti}$
  - U_{0i} → A random intercept for differences in person means, assumed uncorrelated with equal variance across persons → "**BP (mean) variation**" = $\tau^2_{U_0}$ is now "leftover" after predictors
  - e_{ti} → A residual that represents remaining time-to-time variation, usually assumed uncorrelated with equal variance across observations (now, persons and time) → "**WP variation**" = σ^2_e is also now "leftover" after predictors

Repeated Measures (RM) ANOVA

- Models with a random intercept to predict a constant correlation of outcomes from the same person are also called:
  - Hierarchical linear models, multilevel models, general linear mixed models, growth curve models, random coefficient models…
- The key to how these latter models can extend beyond traditional RM ANOVA lies in maximum likelihood estimation (ML in MLM) instead of least squares (LS in ANOVA)
  - Options for other types of outcomes: Normal vs. not-normal
  - Options for uncooperative participants: Missing or unbalanced data
  - Options for extension: What if a random intercept is not enough to describe all sources of between-person differences?

Addressing Uncooperative Participants

- ML allows **incomplete** and **unbalanced** responses…

RM ANOVA via LS: uses **multivariate** (wide) data structure:

ID	Sex	T1	T2	T3	T4
100	0	5	6	8	12
101	1	4	7	.	11

People missing any data are excluded (data from ID 101 are not included at all)

MLM via ML: uses **stacked** (long) data structure:

Only occasions with missing data are excluded

ID 100 uses 4 cases
ID 101 uses 3 cases

ID	Sex	Time	Y
100	0	1	5
100	0	2	6
100	0	3	8
100	0	4	12
101	1	1	4
101	1	2	7
101	1	3	.
101	1	4	11

Time can also be **unbalanced** across people such that each person can have his or her own measurement schedule: Time "0.9" "1.4" "3.5" "4.2"…

Moving Beyond a Random Intercept

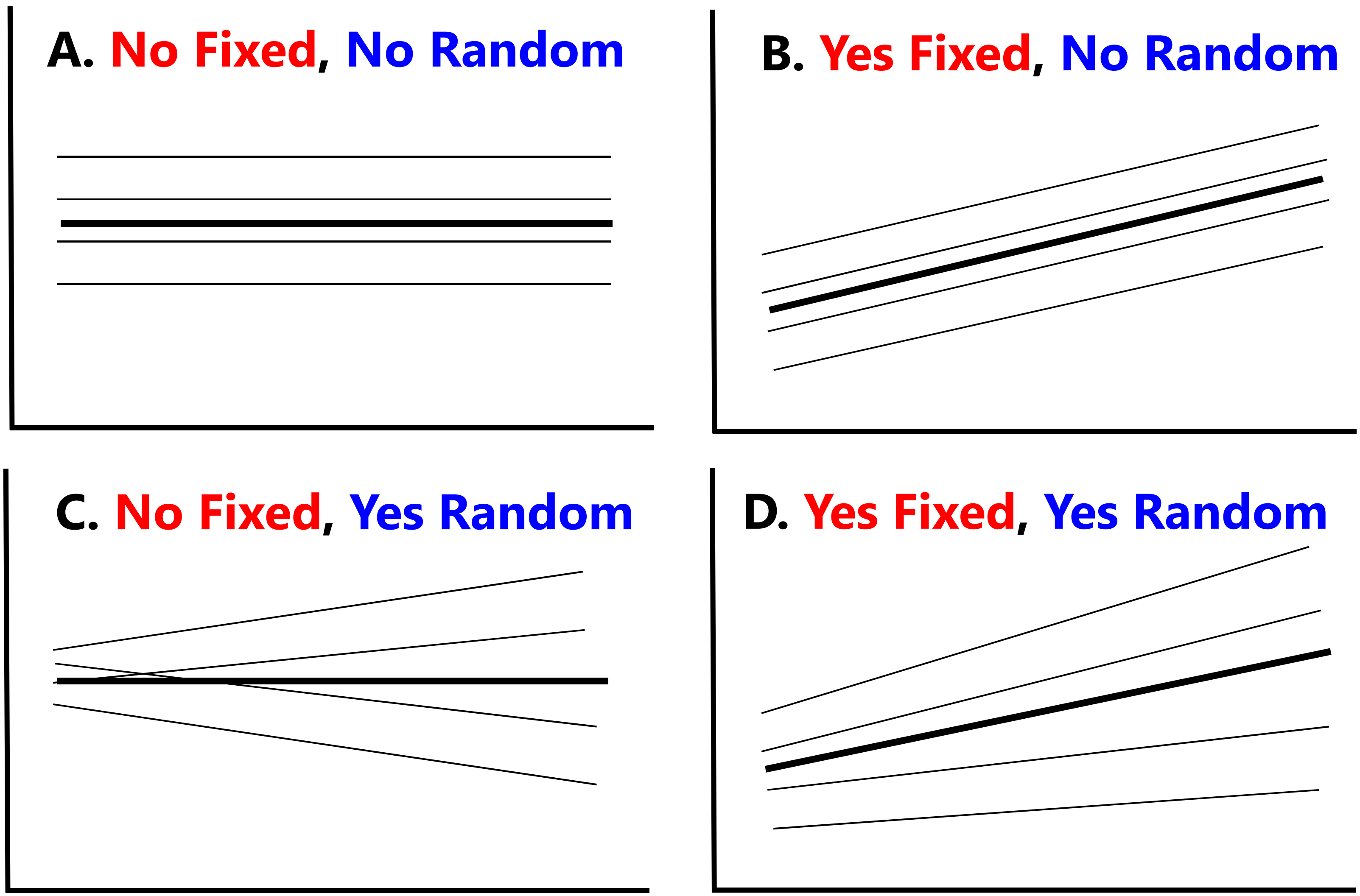
- 2 questions about the possible effects of *time*:

1. **Is there an effect of time on average?**
   - If the line describing the sample means not flat?
   - Significant **FIXED** effect of time

2. **Does the average effect of time vary across individuals?**
   - Does each individual need his or her own line?
   - Significant **RANDOM** effect of time

Fixed and Random Effects of **Time**

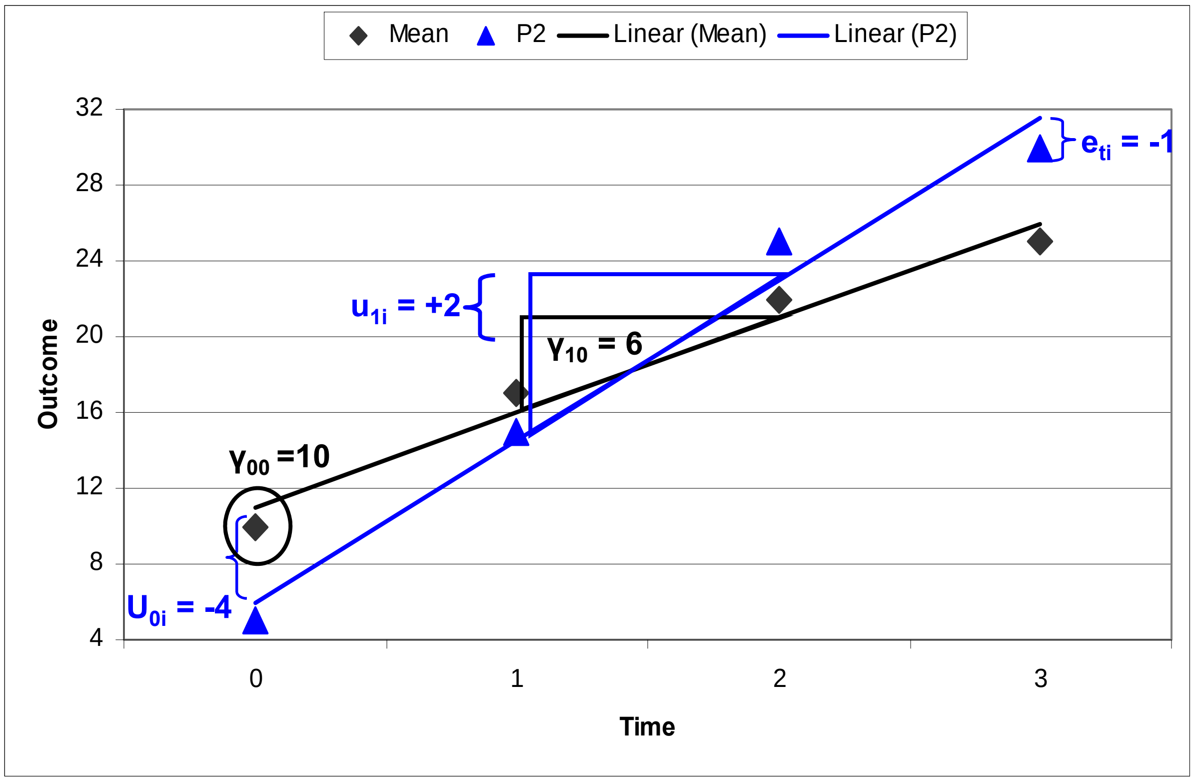
(Note: The intercept is random in every figure)



A "Random Linear Time" Model

$$y_{ti} = (\gamma_{00} + U_{0i}) + (\gamma_{10} + U_{1i})(Time_{ti}) + e_{ti}$$

- γ_{00}: Fixed Intercept
- U_{0i}: Random Intercept Deviation
- γ_{10}: Fixed Slope
- U_{1i}: Random Slope Deviation
- e_{ti}: error for person i at time t



6 Parameters:

2 Fixed Effects:

γ_{00} Intercept, γ_{10} Slope

2 Random Effects Variances:

U_{0i} Intercept Variance $= \tau^2_{U_0}$

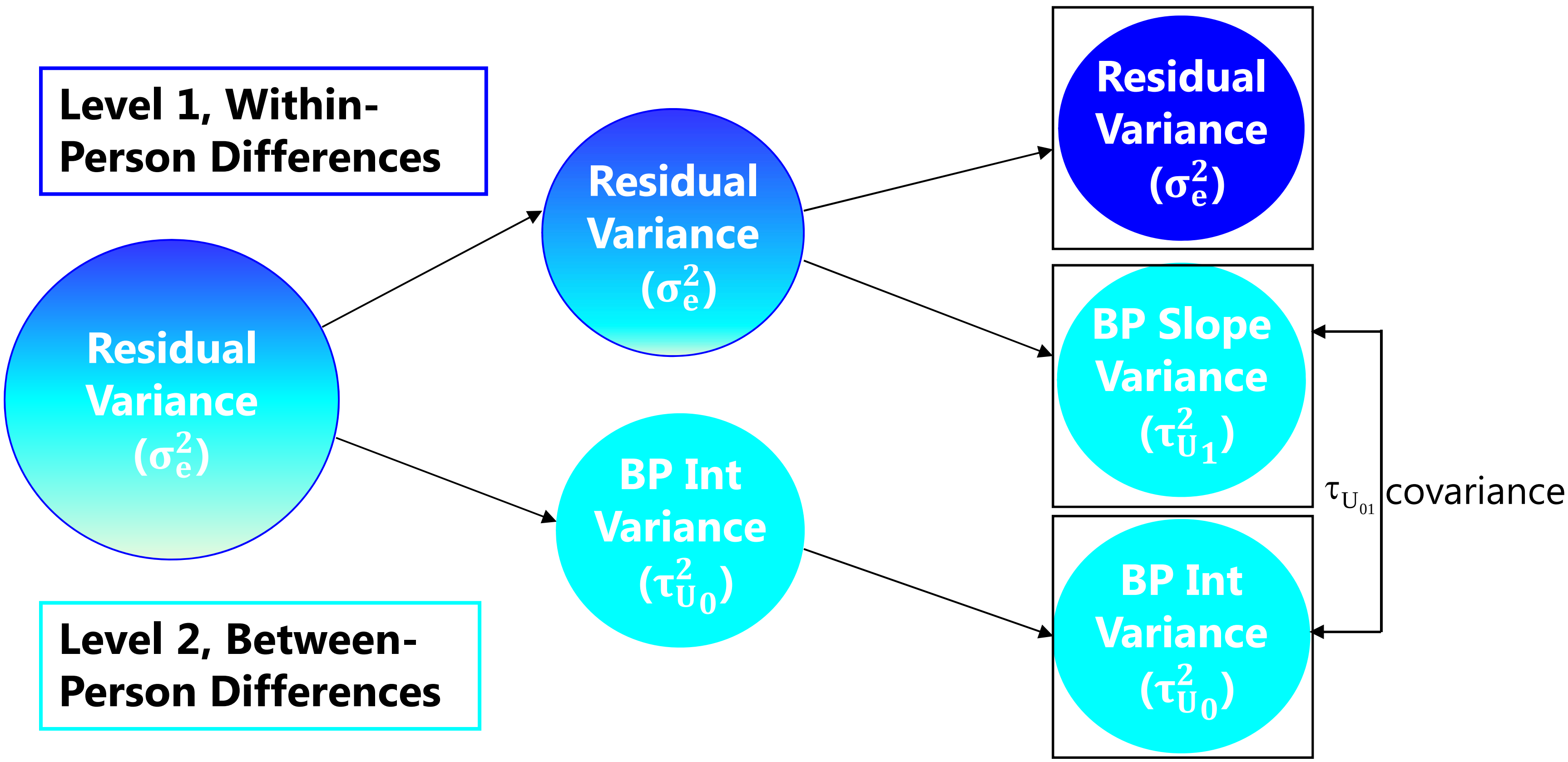
U_{1i} Slope Variance $= \tau^2_{U_1}$

Int-Slope Covariance $= \tau_{U_{01}}$

1 e_{ti} Residual Variance $= \sigma^2_e$

MLM "Handles" Dependency

- Where does each kind of person dependency go? Into a new random effects variance component (or "pile" of variance):



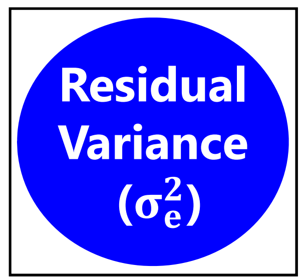
Piles of Variance (as Random Effects)

- By adding a random slope, we **carve up** our total variance into 3 piles:
  - BP (error) variance around intercept
  - BP (error) variance around slope
  - WP (error) residual variance

These 2 piles are just 1 pile of "error variance" in Univariate RM ANOVA

- **But making piles does NOT make error variance go away…**

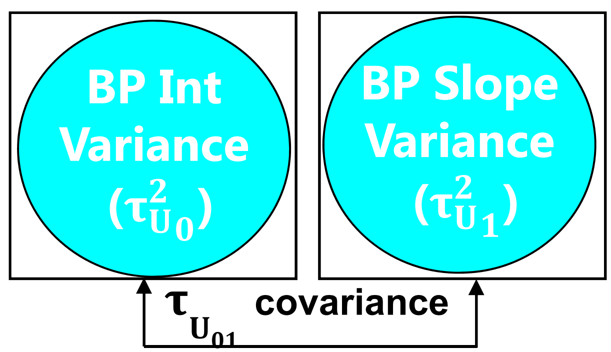
Level 1 (one source of) <u>Within-Person Variation</u>: gets accounted for by time-level predictors



FIXED effects make variance go away (explain variance).

RANDOM effects just make a new pile of variance.

Level 2 (two sources of) <u>Between-Person Variation</u>: gets accounted for by person-level predictors



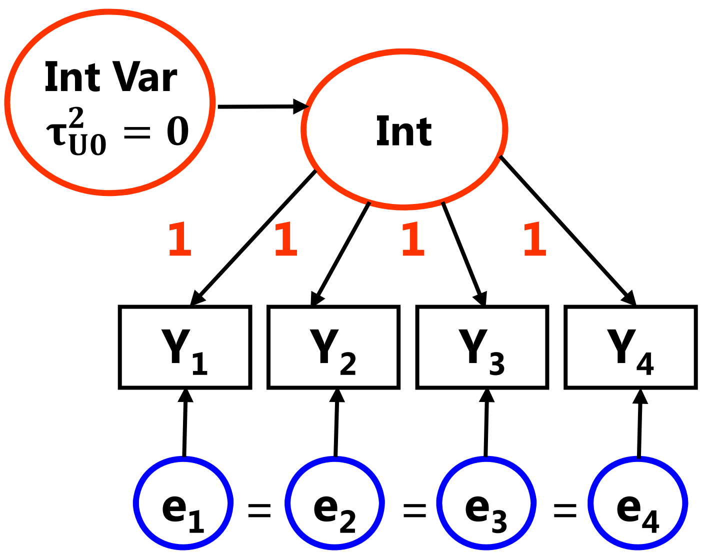
Options for Longitudinal Models

- Although models and software are separate, longitudinal data can be analyzed via multiple analytic frameworks:
  - **"Multilevel/Mixed Models"**
    - Dependency over time, persons, groups, etc. are modeled via random effects (multivariate → univariate through "levels" of stacked/long data)
    - Builds on GLM, generalizes more easily to additional levels of analysis and crossed sampling (e.g., if people change groups over time)
  - **"Structural Equation Models"**
    - Dependency over time *only* is modeled via latent variables (single-level analysis using multivariate/wide data)
    - Generalizes easier to broader analysis of latent constructs, mediation, and multivariate multilevel models in general (aka, "Multilevel SEM")

  - Because random effects and latent variables are the same thing, many longitudinal models can be specified/estimated either way

Random Effects as Latent Variables

- **BP model: e_{ti}-only model for the variance**
  - **$y_{ti} = \gamma_{00} + e_{ti}$**



Mean of the intercept factor = fixed intercept γ_{00}

Loadings of intercept factor = 1 (all occasions contribute equally)

Item intercepts = 0 (always)

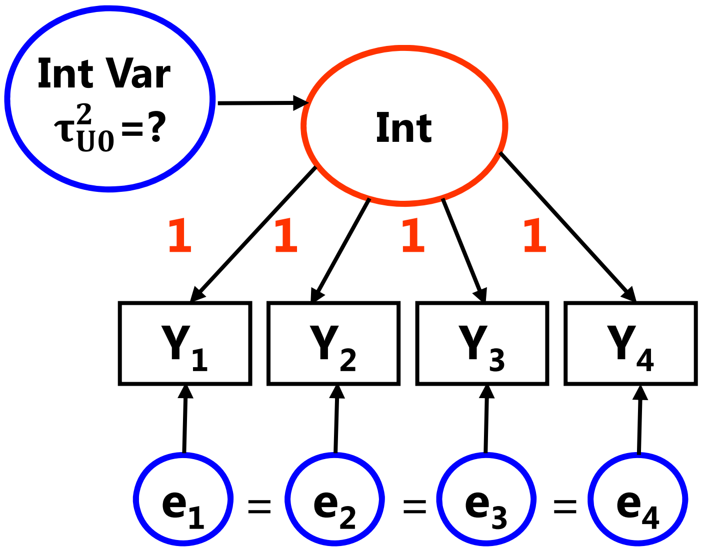
Variance of intercept factor = 0 so far

Residual variance (e) is assumed to be equal across occasions

  - After controlling for the *fixed* intercept, residuals are assumed uncorrelated

Random Effects as Latent Variables

- **+WP model: U_{0i} + e_{ti} model for the variance**
  - **$y_{ti} = \gamma_{00} + U_{0i} + e_{ti}$**



Mean of the intercept factor = fixed intercept γ_{00}

Loadings of intercept factor= 1 (all occasions contribute equally)

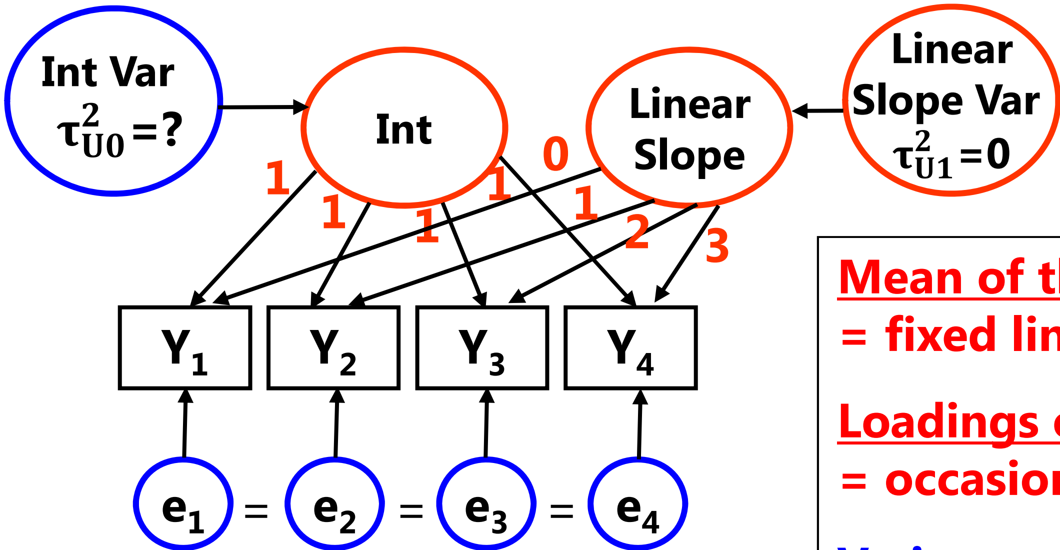
Variance of intercept factor = random intercept variance

Residual variance (e) is assumed to be equal across occasions

  - After controlling for the *random* intercept, residuals are assumed uncorrelated

Random Effects as Latent Variables

- **Fixed linear time, random intercept model:**
  - $y_{ti} = \gamma_{00} + (\gamma_{10}Time_{ti}) + U_{0i} + e_{ti}$



Mean of the linear slope factor = fixed linear slope γ_{10}

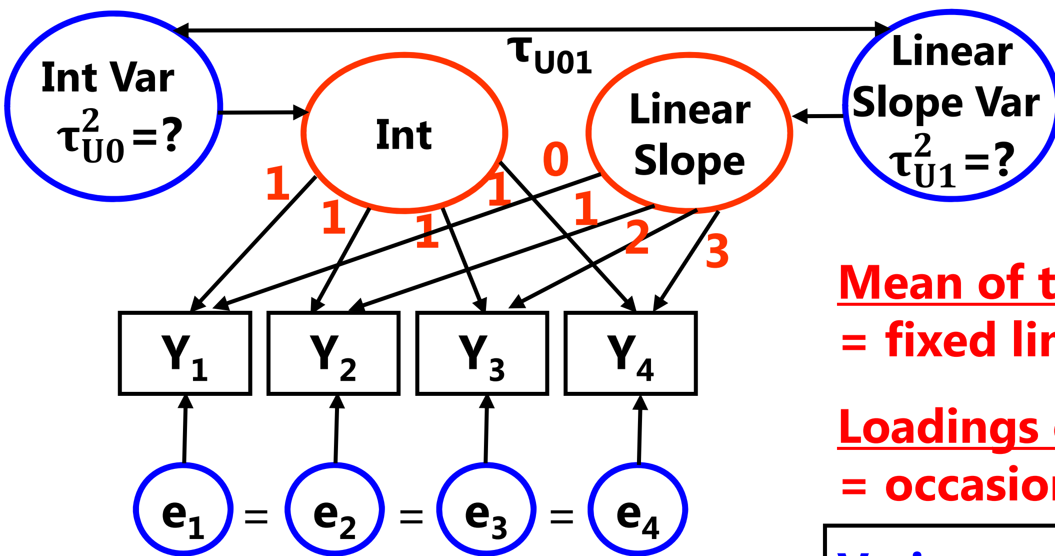
Loadings of linear slope factor = occasions (keep real time)

Variance of linear slope factor = 0

- After controlling for the *fixed linear slope and random intercept*, residuals are assumed uncorrelated

Random Effects as Latent Variables

- **Random linear time model:**
  - $y_{ti} = \gamma_{00} + (\gamma_{10}\text{Time}_{ti}) + U_{0i} + (U_{1i}\text{Time}_{ti}) + e_{ti}$



Mean of the linear slope factor = fixed linear slope γ_{10}

Loadings of linear slope factor = occasions (keep real time)

Variance of linear slope factor = random slope variance

  - After controlling for the *random linear slope and random intercept*, residuals are assumed uncorrelated

Intermediate Summary

- Longitudinal models use random effects/latent variables to quantify and predict sources of variability:
  - Between persons (BP) in intercept and (aspects of) change over time
    - Why do people start at different places?
    - Why do people change at different rates?
  - Within persons (WP) after controlling for individual change
    - Why are you off your line today?

- Individuals are conceptualized as **continuously varying** from one another in each of the between-person dimensions
  - If so, then one set of variances describes the entire sample
  - What if that's not the case?
  - Enter the "heterogeneous population model" variants…

Models for Finding "Hidden" Groups

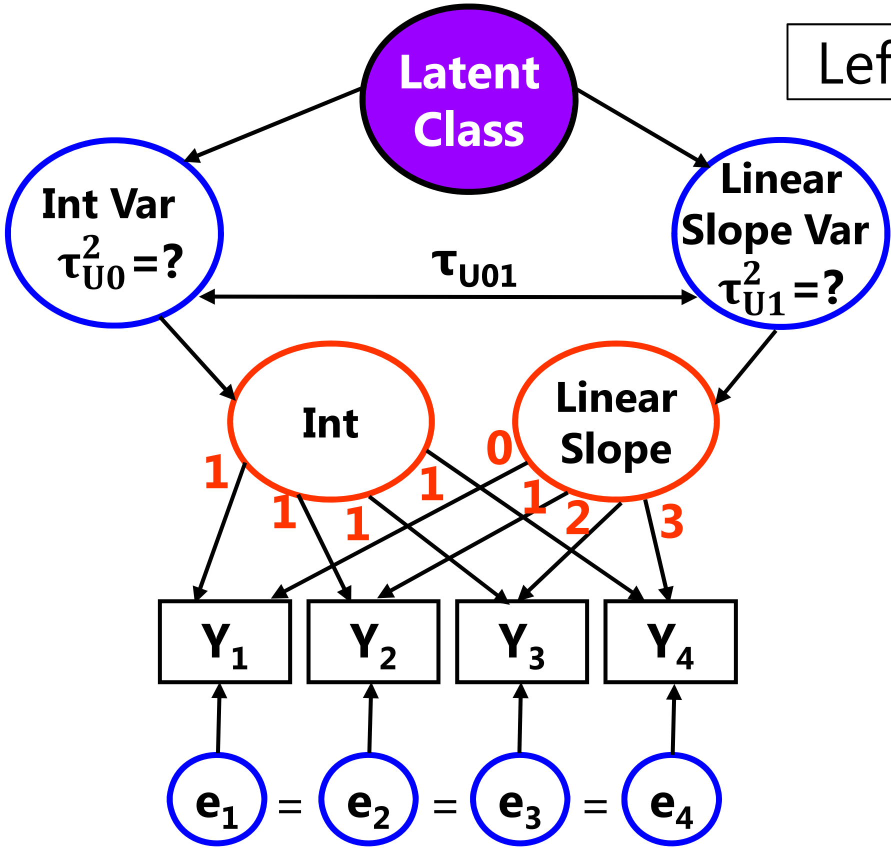
- Related to traditional cluster analysis (using least squares)
  - ML variants were popularized by Lazarsfeld and Henry (1968)

- Instead of continuous individual variation, models postulate existence of qualitatively different latent (hidden) subgroups

- More generally known as "finite mixture models," specific model names depend on type of outcomes to be classified:
  - Categorical, cross-sectional outcomes? "Latent class analysis"
  - Continuous, cross-sectional outcomes? "Latent profile analysis"
  - Change in group status over time? "Latent transition analysis"
  - Change in longitudinal outcomes over time? "Growth mixture models"
  - All have similar limitations, but we'll focus on **growth mixture models**

Growth Mixture Models (GMM)

Left: typical depiction of a GMM



- **Latent Class** = categorical unobserved variable that predicts probabilistic membership in c classes

GMMs are advertised being able to detect differing latent trajectories across people, but as used in practice, they have significant limitations:

1. Completely exploratory
2. Sensitive to non-normality
3. Distort individual variability
4. Classes can only predict existing random effects
5. Classes are not needed to examine prediction

1. GMMs are exploratory

- **How many classes?** ***????***
  - Programs provide relative goodness of fit info, but simulation results suggest these are problematic in practice
    - Information criteria (AIC, BIC) are inadequate for determining # classes
    - Entropy based on classification is only valid if the model fits…
- **What are the classes? How should they differ?** ***????***
  - Nature of the classes is determined entirely by the program
  - Get probability of membership to each class for each person, but this will likely change after predicting class membership
    - Should NEVER use the most likely class as an observed variable!

2. GMMs predict non-normality

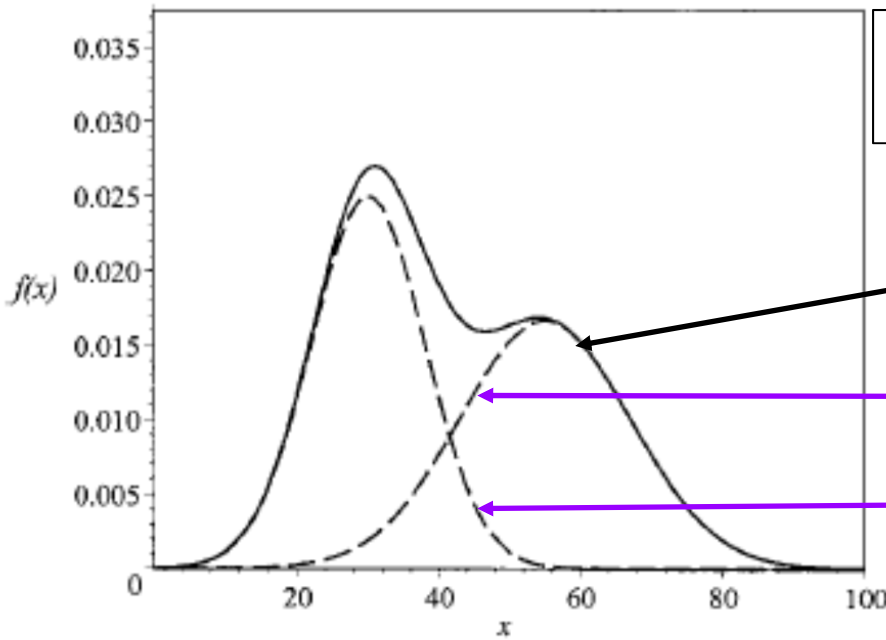


Figure 2 from Bauer & Curran (2003, *Psychological Methods*)

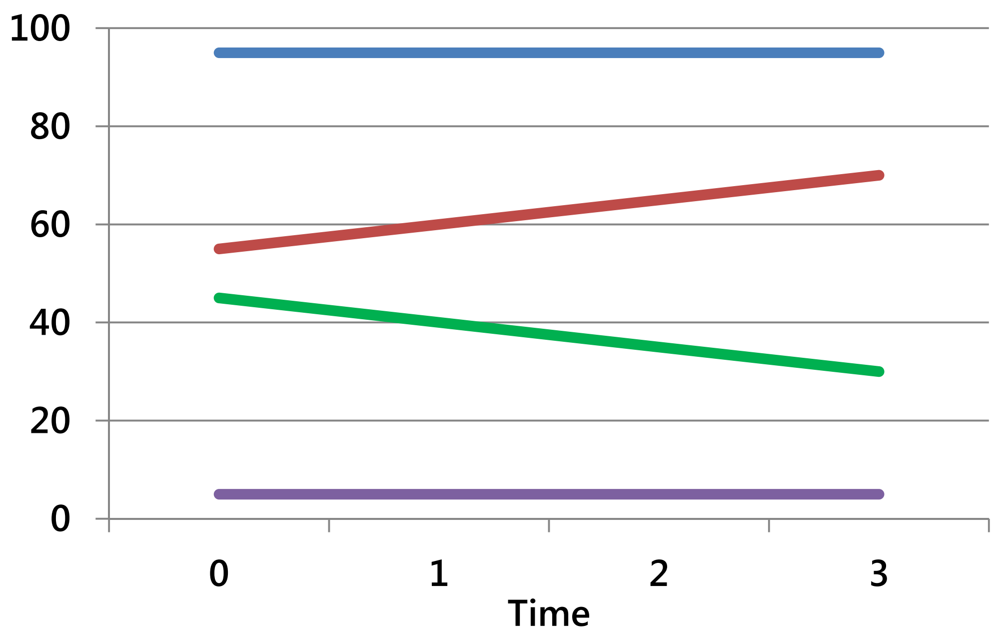
The overall non-normal distribution in sample…

… is described by a mixture of two normal distributions instead

- A lesser-known but statistically indistinguishable purpose of GMMs is to approximate a non-normal overall distribution
  - **So if you fit a GMM erroneously assuming conditional normality, you WILL find two or more latent classes for that reason alone**

3. GMMs distort individual variability

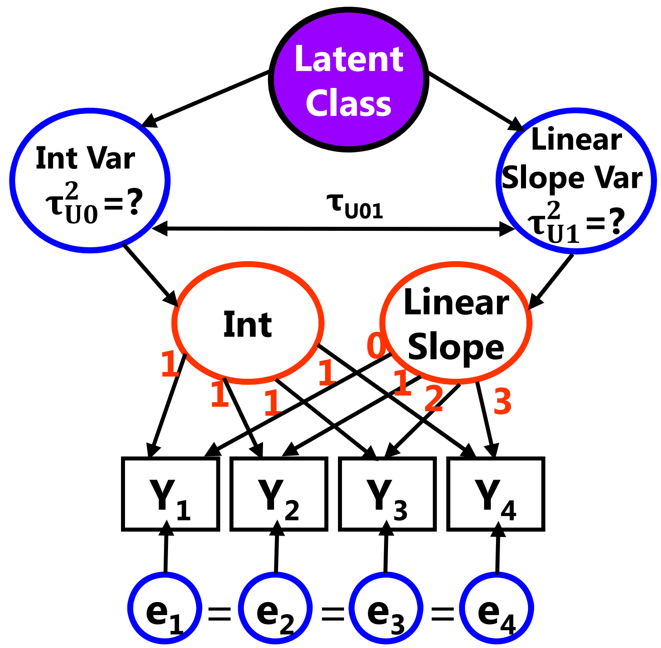
- What about individual differences within classes?
  Well, that depends on the program, too:
  - SAS PROC TRAJ: What variability?
  - Mplus: Variability is equal across classes, which is likely to be logically impossible… (but freeing this constraint leads to estimation problems)

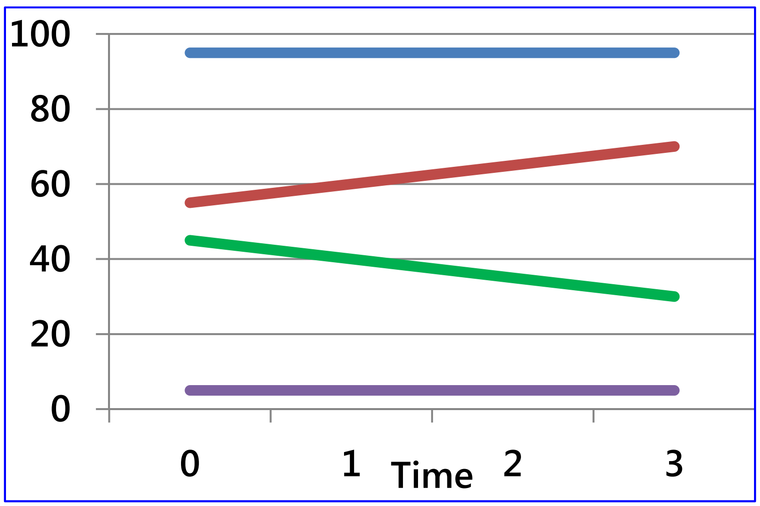


For example: The "high stable" and "low stable" groups cannot possibly have the same intercept and slope variability as the other groups…

4. GMMs can only predict model-specified random effects

- Latent classes serve to categorize existing intercepts and slopes…
  - For example, given the specification of a random linear time slope model, latent groups may only differ in level and kinds of linear change…

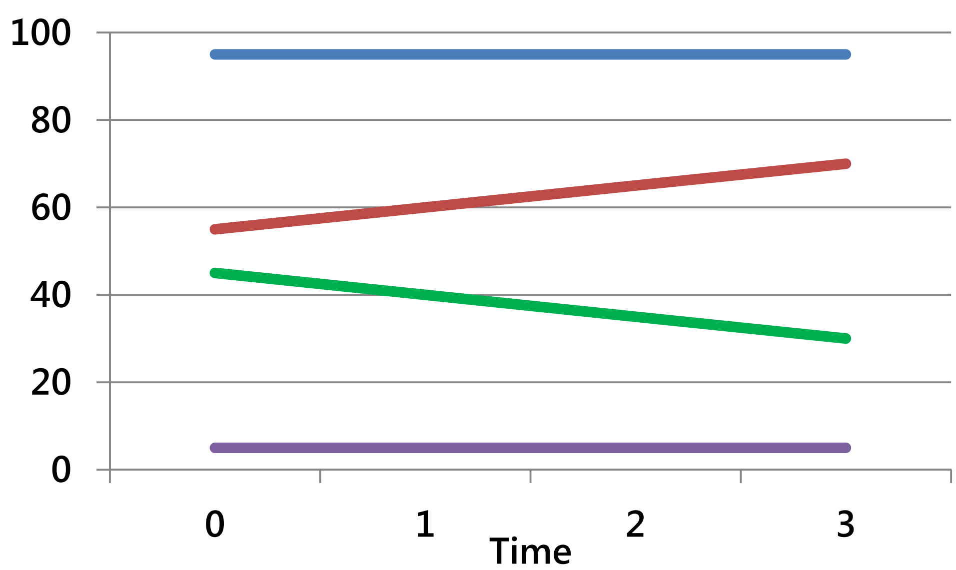


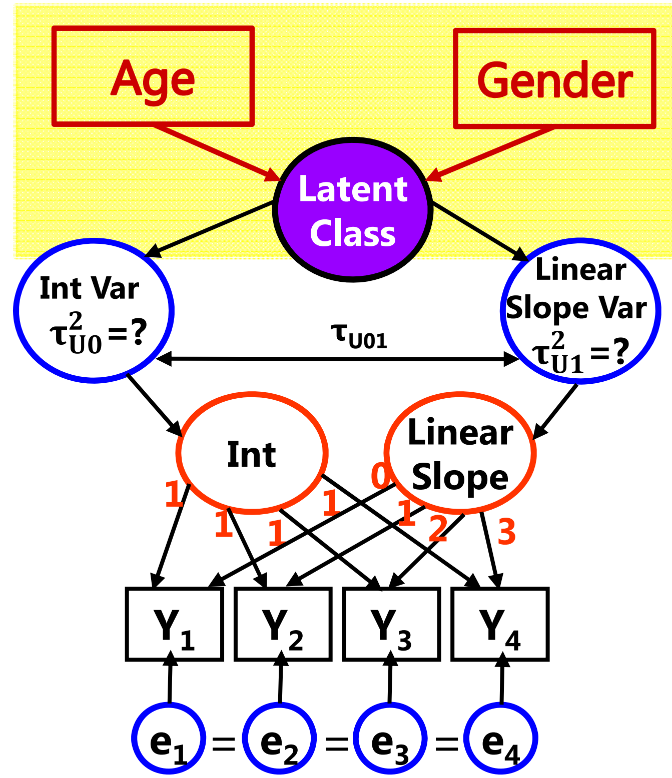


… just as people *already* do in the random linear growth model!

Latent classes just get in the way.

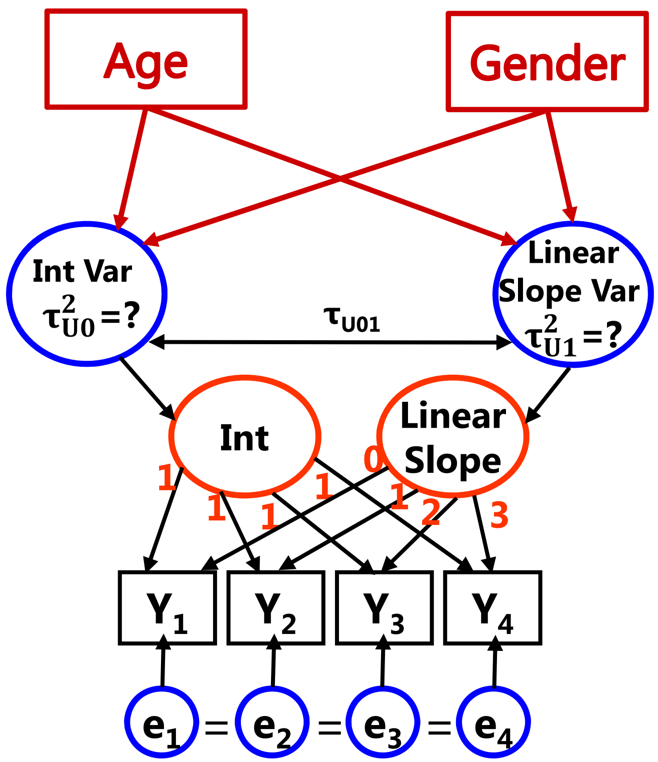
5. GMMs are not needed to examine prediction

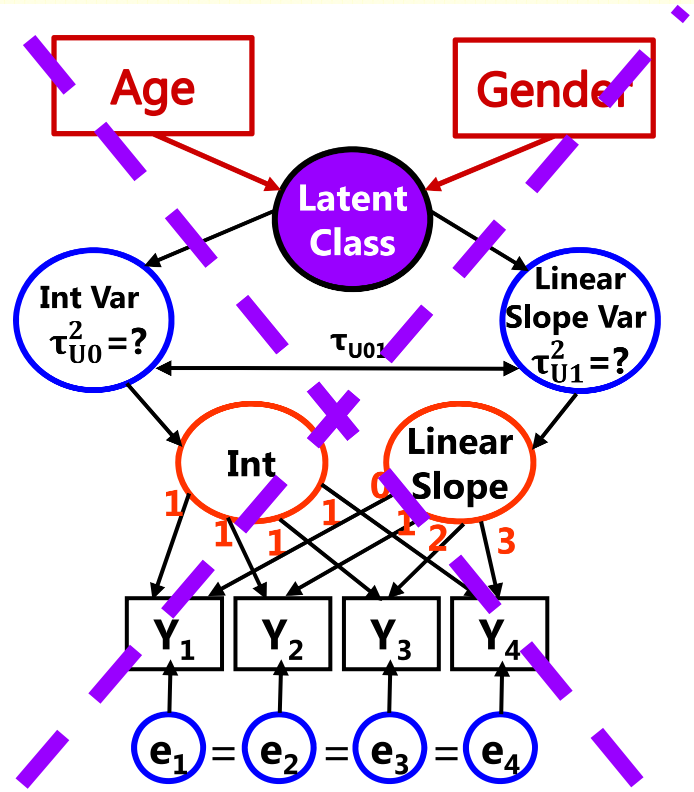




- After fitting a GMM, it is often of interest to then **predict class membership from covariates**…

5. GMMs are not needed to examine prediction





- …but the covariates should directly predict the random intercept and slopes themselves instead!

So what should we do instead?

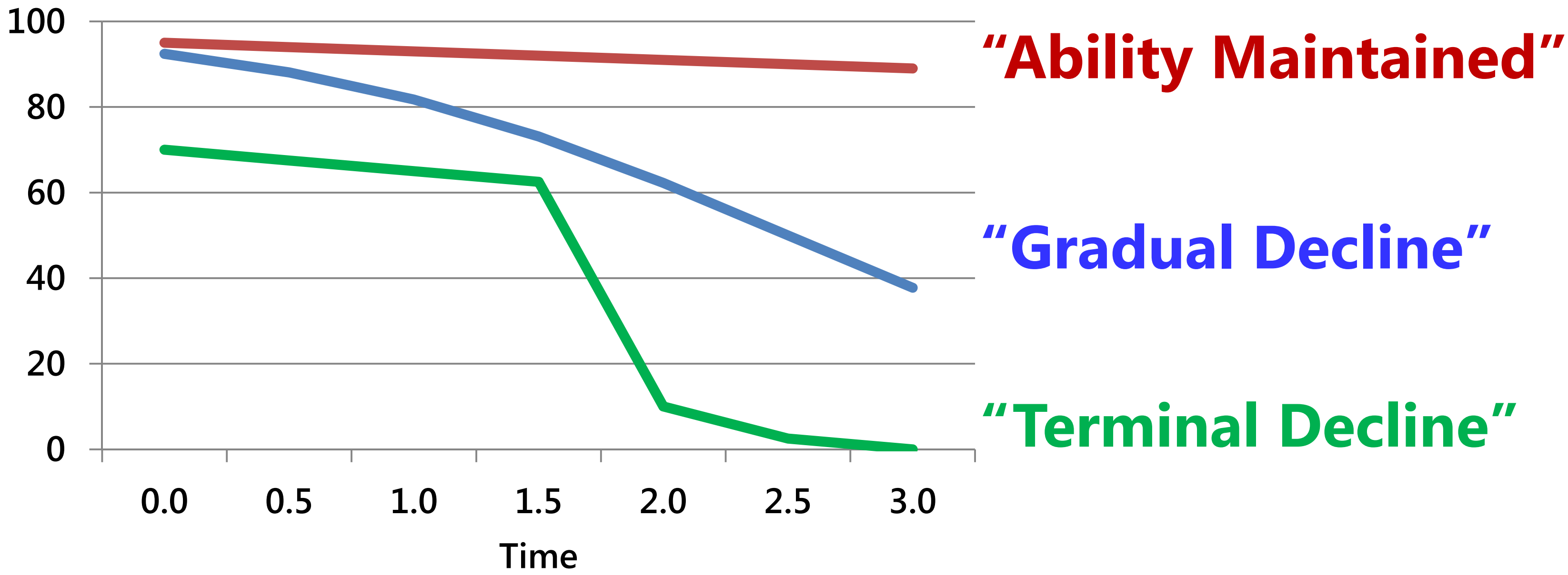
- Before fitting a typical GMMs, specify the most **appropriate conditional outcome distribution**
  - Account for floor/ceiling effects of observed measures

- Determine if groups are really necessary to answer your questions… for prediction of differences, probably not!

- If differences due to known predictors are of interest, consider **location-scale longitudinal models** instead (i.e., heterogeneous variance models; see Don Hedeker's work)
  - Allows for prediction of mean differences in intercepts and slopes, as well as prediction of differences in their **amount of variability**
  - Also very useful to intra-individual variability (IIV) designs

So what should we do instead?

- Specify a **confirmatory, hypothesis-driven model** that defines the different group trajectories *a priori*
- Most useful given **qualitatively different** kinds of change
  - Fit different model of change within each group
  - Constrain parameters as needed to ensure order/interpretability



Conclusions

- Longitudinal models with random effects/latent variables expand on traditional RM ANOVA via ML:
  - Multiple sources of between-person differences (from random intercepts only to random slopes for change)
  - Individuals vary continuously from another in growth terms

- Typical uses of growth mixture models try to describe these continuous differences via latent groups instead
  - But are completely exploratory, sensitive to violations of distributional assumptions, inflexible with respect to forms of change, and get in the way of predicting individual differences
  - Confirmatory models may remedy these problems but are seldom used in longitudinal applications