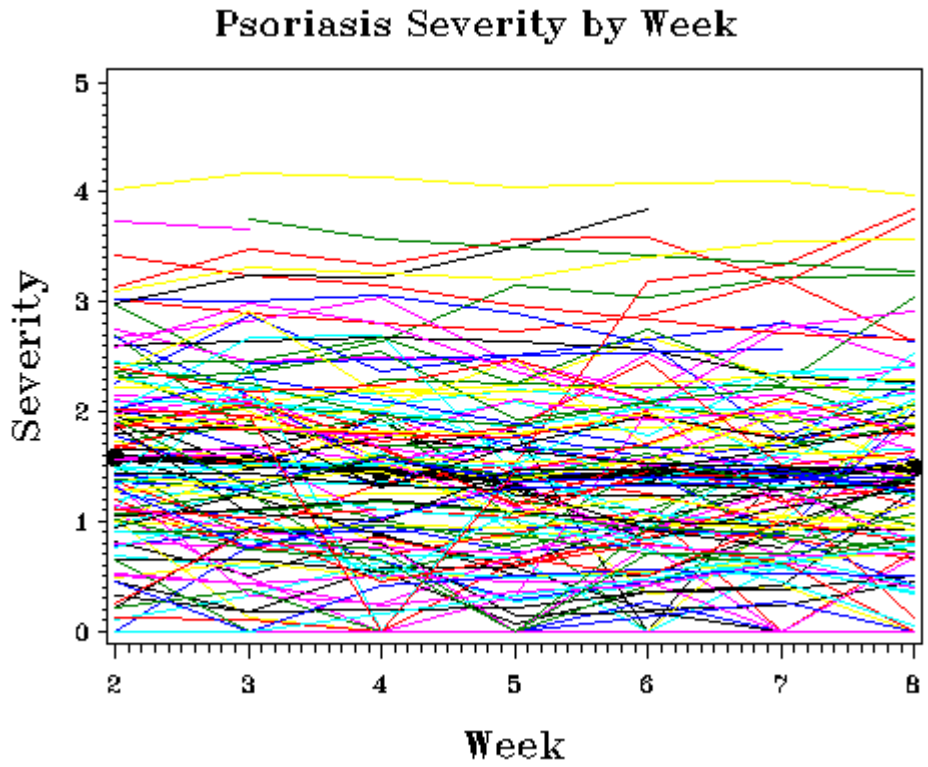
Example 2: Within-Person Fluctuation in Symptom Severity over Time
(complete syntax and output available for SAS, SPSS, and STATA electronically)



These data come from a study of weekly fluctuation in psoriasis severity. There was no intervention and no real reason to expect systematic growth (as shown by the mostly flat solid black line for the means).

But we still need to be concerned about the model for the variances. Specifically, the variances across occasions may need to differ, and the covariances across occasions may need to differ as well.

We will begin by including just a fixed intercept in the model for the means, and examine different ACS models for the variances instead.

To begin, let's see what the observed pattern of variances and covariances over time looks like by estimating an *n*-order unstructured **R** baseline model for the variances → each variance and covariance estimated is separately, with no constraints, so this is a description, not really a model. Note: you can only estimate an unstructured model if time is balanced across persons and you have more people than parameters estimated!

_n_-order Unstructured R-only Model: the answer key of all possible variances and covariances

```
TITLE1 "SAS n-order Unstructured R Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity =  / SOLUTION DDFM=Satterthwaite;
       REPEATED week / TYPE=UN R RCORR SUBJECT=subid;
RUN; TITLE1;


TITLE "SPSS n-order Unstructured R-only Model".
MIXED severity BY subid week
   /METHOD = REML
   /PRINT = SOLUTION TESTCOV R
   /FIXED =
   /REPEATED = week | SUBJECT(subid) COVTYPE(UN).
TITLE.


 * STATA n-order Unstructured R-only Model
xtmixed severity ,  || subid:  , noconstant  ///
       variance reml residuals(unstructured,t(Week)),
       estat ic, n(124),   // get AIC and BIC
```

SAS Output:

Note that given *n*=7, this model requires *n**(*n*+1) / 2 = 28 covariance parameters!! You'd need at least 28 people to estimate it (and it may not be possible even then).

Dimensions

Covariance Parameters	28	Cov Parms = total number of parameters in model for variances
Columns in X	1	Columns in X = total number of fixed effects plus intercept
Columns in Z	0	Columns in Z = total number of U's (not counting covariances)
Subjects	124	Subjects = number of persons in level 2 (with at least 1 obs)
Max Obs Per Subject	7	Max Obs Per Subject = max number of time points per person

Number of Observations

Number of Observations Read	770	
Number of Observations Used	770	
Number of Observations Not Used	0	Make sure to keep track of how many cases get dropped!!

Estimated R Matrix for SUBID 100 → TOTAL variance and covariance → ANSWER KEY

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.7477**	0.7190	0.6817	0.6807	0.6124	0.6182	0.6328
2	0.7190	**0.8266**	0.7428	0.7475	0.6571	0.6619	0.6541
3	0.6817	0.7428	**0.8346**	0.7498	0.6524	0.6390	0.6172
4	0.6807	0.7475	0.7498	**0.8366**	0.7292	0.7206	0.6805
5	0.6124	0.6571	0.6524	0.7292	**0.8285**	0.7280	0.7313
6	0.6182	0.6619	0.6390	0.7206	0.7280	**0.7911**	0.7352
7	0.6328	0.6541	0.6172	0.6805	0.7313	0.7352	**0.8464**

Estimated R Correlation Matrix for SUBID 100 → TOTAL correlation → answer key

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.9146	0.8630	0.8608	0.7781	0.8038	0.7955
2	**0.9146**	1.0000	0.8943	0.8989	0.7940	0.8185	0.7820
3	**0.8630**	**0.8943**	1.0000	0.8974	0.7846	0.7864	0.7343
4	**0.8608**	**0.8989**	**0.8974**	1.0000	0.8760	0.8859	0.8088
5	**0.7781**	**0.7940**	**0.7846**	**0.8760**	1.0000	0.8993	0.8734
6	**0.8038**	**0.8185**	**0.7864**	**0.8859**	**0.8993**	1.0000	0.8985
7	**0.7955**	**0.7820**	**0.7343**	**0.8088**	**0.8734**	**0.8985**	1.0000

Covariance Parameter Estimates table is too big to print (has 28 entries!)...

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
893.1	**28**	**949.1**	951.3	981.2	**1028.1**	1056.1

The fit statistics for this *n*-order unstructured **R**-only model will serve as a baseline with which to compare more parsimonious models for the variances.

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1.5178	0.07429	122	20.43	<.0001

Ok, so this is what we are trying to reproduce… next we will try a few **R**-only models and see how they fit.

Here is the most basic model for the variances: an E-only **R**-only model, which assumes equal variance over persons and occasions and that all observations are independent—it is a cross-sectional model.

E-only R-only Model: equal total variances and no total covariance across weeks whatsoever

```
TITLE1 "SAS E-only R-only Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity = / SOLUTION DDFM=Satterthwaite;
       REPEATED week / TYPE=VC R RCORR SUBJECT=subid; RUN; TITLE1;

TITLE "SPSS E-only R-only Model".
MIXED severity BY subid week
   /METHOD = REML
   /PRINT = SOLUTION TESTCOV R
   /FIXED =
   /REPEATED = week | SUBJECT(subid) COVTYPE(ID).
TITLE.

 * STATA E-only R-only Model
xtmixed severity ,  || subid:  , noconstant  ///
       variance reml residuals(independent,t(Week)),
       estat ic, n(124),   // get AIC and BIC
```

SAS Output:

Estimated R Matrix for SUBID 100 = TOTAL variance

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	0.7820						
2		0.7820					
3			0.7820				
4				0.7820			
5					0.7820		
6						0.7820	
7							0.7820

Estimated R Correlation Matrix for SUBID 100

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000						
2		1.0000					
3			1.0000				
4				1.0000			
5					1.0000		
6						1.0000	
7							1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
WEEK	SUBID	0.7820	0.03988	19.61	<.0001	the total variance (equal over occasions)

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
1999.9	1	2001.9	2001.9	2003.0	2004.7	2005.7

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	
Intercept	1.4583	0.03187	769	45.76	<.0001	fixed intercept → here, the grand mean

Compound Symmetry R-only Model: equal total variances and covariances across weeks

```
TITLE1 "SAS Compound Symmetry R-only Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity = / SOLUTION DDFM=Satterthwaite;
       REPEATED week / TYPE=CS R RCORR SUBJECT=subid; RUN; TITLE1;

TITLE "SPSS Compound Symmetry R-only Model".
MIXED severity BY subid week
   /METHOD = REML
   /PRINT = SOLUTION TESTCOV R
   /FIXED =
   /REPEATED = week | SUBJECT(subid) COVTYPE(CS).
TITLE.

 * STATA Compound Symmetry R-only Model
xtmixed severity ,  || subid:  , noconstant  ///
       variance reml residuals(exchangeable,t(Week)),
       estat ic, n(124),   // get AIC and BIC
```

SAS Output:

Estimated R Matrix for SUBID 100 → TOTAL variance and covariance

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	0.8127	0.6820	0.6820	0.6820	0.6820	0.6820	0.6820
2	0.6820	0.8127	0.6820	0.6820	0.6820	0.6820	0.6820
3	0.6820	0.6820	0.8127	0.6820	0.6820	0.6820	0.6820
4	0.6820	0.6820	0.6820	0.8127	0.6820	0.6820	0.6820
5	0.6820	0.6820	0.6820	0.6820	0.8127	0.6820	0.6820
6	0.6820	0.6820	0.6820	0.6820	0.6820	0.8127	0.6820
7	0.6820	0.6820	0.6820	0.6820	0.6820	0.6820	0.8127

Estimated R Correlation Matrix for SUBID 100 → TOTAL correlation

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.8393	0.8393	0.8393	0.8393	0.8393	0.8393
2	0.8393	1.0000	0.8393	0.8393	0.8393	0.8393	0.8393
3	0.8393	0.8393	1.0000	0.8393	0.8393	0.8393	0.8393
4	0.8393	0.8393	0.8393	1.0000	0.8393	0.8393	0.8393
5	0.8393	0.8393	0.8393	0.8393	1.0000	0.8393	0.8393
6	0.8393	0.8393	0.8393	0.8393	0.8393	1.0000	0.8393
7	0.8393	0.8393	0.8393	0.8393	0.8393	0.8393	1.0000

The intraclass correlation in RCORR can be computed as:

$$\text{ICC} = \frac{0.6820}{0.6820 + 0.1306} = .84$$

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
CS	SUBID	0.6820	0.09019	7.56	<.0001	compound symmetry parameter
Residual		0.1306	0.007270	17.97	<.0001	residual variance (Var of e's)

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
1049.7	2	1053.7	1053.7	1056.0	1059.4	1061.4

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	950.14	<.0001

Is this CS model better than the e-only model?
$-2\Delta LL(1) = 950.2$, $p < 001$
Note that the ICC is given in the RCORR matrix (.8393).

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1.4767	0.07548	123	19.56	<.0001

Auto-Regressive R-only Model: equal total variances and an AR1 total correlation across weeks

```
TITLE1 "SAS Auto-Regressive R-only Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity = / SOLUTION DDFM=Satterthwaite;
       REPEATED week / TYPE=AR(1) R RCORR SUBJECT=subid; RUN; TITLE1;

TITLE "SPSS Auto-Regressive R-only Model".
MIXED severity BY subid week
   /METHOD = REML
   /PRINT = SOLUTION TESTCOV R
   /FIXED =
   /REPEATED = week | SUBJECT(subid) COVTYPE(AR1).
TITLE.

 * STATA Auto-Regressive R-only Model
xtmixed severity ,  || subid:  , noconstant  ///
       variance reml residuals(ar1,t(Week)),
       estat ic, n(124),   // get AIC and BIC
```

SAS Output:

Estimated R Matrix for SUBID 100 → TOTAL variance and covariance

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.7999**	0.7163	0.6415	0.5745	0.5145	0.4607	0.4126
2	0.7163	**0.7999**	0.7163	0.6415	0.5745	0.5145	0.4607
3	0.6415	0.7163	**0.7999**	0.7163	0.6415	0.5745	0.5145
4	0.5745	0.6415	0.7163	**0.7999**	0.7163	0.6415	0.5745
5	0.5145	0.5745	0.6415	0.7163	**0.7999**	0.7163	0.6415
6	0.4607	0.5145	0.5745	0.6415	0.7163	**0.7999**	0.7163
7	0.4126	0.4607	0.5145	0.5745	0.6415	0.7163	**0.7999**

Estimated R Correlation Matrix for SUBID 100 → TOTAL correlation

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.8955	0.8020	0.7182	0.6432	0.5760	0.5158
2	**0.8955**	1.0000	0.8955	0.8020	0.7182	0.6432	0.5760
3	**0.8020**	**0.8955**	1.0000	0.8955	0.8020	0.7182	0.6432
4	**0.7182**	**0.8020**	**0.8955**	1.0000	0.8955	0.8020	0.7182
5	**0.6432**	**0.7182**	**0.8020**	**0.8955**	1.0000	0.8955	0.8020
6	**0.5760**	**0.6432**	**0.7182**	**0.8020**	**0.8955**	1.0000	0.8955
7	**0.5158**	**0.5760**	**0.6432**	**0.7182**	**0.8020**	**0.8955**	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
AR(1)	SUBID	0.8955	0.01148	78.01	<.0001	Total Auto-regressive correlation
Residual		0.7999	0.08044	9.94	<.0001	Total variance (equal across weeks)

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
990.9	2	994.9	994.9	997.2	1000.5	1002.5

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1.5148	0.07031	129	21.54	<.0001

AR1 also forces all variances equal, but with correlations declining sharply with time (r, r^2, r^3... = .8955, .8020, .7182…). Relative to the UN model, the BIC (but not the AIC) is happier.

But the AR1 model fits significantly worse than the UN model, $-2\Delta LL(26) = 97.8$, $p < .001$.

Toeplitz(*n*) R-only Model: equal total variances and 6 lagged total covariances across weeks

```
TITLE1 "SAS Toeplitz (n=7 bands) R-only Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity = / SOLUTION DDFM=Satterthwaite;
       REPEATED week / TYPE=TOEP(7) R RCORR SUBJECT=subid; RUN; TITLE1;
```

```
TITLE "SPSS Toeplitz (n=7 bands) R-only Model".
MIXED severity BY subid week
   /METHOD = REML
   /PRINT = SOLUTION TESTCOV R
   /FIXED =
   /REPEATED = week | SUBJECT(subid) COVTYPE(TP).
TITLE.
```

```
 * STATA Toeplitz (n=7 bands) R-only Model
xtmixed severity ,  || subid:  , noconstant  ///
       variance reml residuals(toeplitz6,t(Week)),
       estat ic, n(124),   // get AIC and BIC
```

For Toeplitz in SAS, (*n*) indicates the total number of bands including the residual variance on the diagonal + 6 covariances for the 7 occasions.

This model did not estimate properly in SPSS, but here is what the code should have been.

For Toeplitz in STATA, (*n*) indicates the number of bands NOT including the residual variance on the diagonal: just 6 covariances for the 7 occasions.

SAS Output:

Estimated R Matrix for SUBID 100 → TOTAL variance and covariance

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.8103**	0.7257	0.6974	0.6576	0.6394	0.6368	0.6541
2	0.7257	**0.8103**	0.7257	0.6974	0.6576	0.6394	0.6368
3	0.6974	0.7257	**0.8103**	0.7257	0.6974	0.6576	0.6394
4	0.6576	0.6974	0.7257	**0.8103**	0.7257	0.6974	0.6576
5	0.6394	0.6576	0.6974	0.7257	**0.8103**	0.7257	0.6974
6	0.6368	0.6394	0.6576	0.6974	0.7257	**0.8103**	0.7257
7	0.6541	0.6368	0.6394	0.6576	0.6974	0.7257	**0.8103**

Estimated R Correlation Matrix for SUBID 100 → TOTAL correlation

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.8955	0.8606	0.8115	0.7890	0.7859	0.8072
2	**0.8955**	1.0000	0.8955	0.8606	0.8115	0.7890	0.7859
3	**0.8606**	**0.8955**	1.0000	0.8955	0.8606	0.8115	0.7890
4	**0.8115**	**0.8606**	**0.8955**	1.0000	0.8955	0.8606	0.8115
5	**0.7890**	**0.8115**	**0.8606**	**0.8955**	1.0000	0.8955	0.8606
6	**0.7859**	**0.7890**	**0.8115**	**0.8606**	**0.8955**	1.0000	0.8955
7	**0.8072**	**0.7859**	**0.7890**	**0.8115**	**0.8606**	**0.8955**	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
TOEP(2)	SUBID	0.7257	0.09004	8.06	<.0001	Total Covariance for t-1
TOEP(3)	SUBID	0.6974	0.08997	7.75	<.0001	Total Covariance for t-2
TOEP(4)	SUBID	0.6576	0.09003	7.30	<.0001	Total Covariance for t-3
TOEP(5)	SUBID	0.6394	0.09019	7.09	<.0001	Total Covariance for t-4
TOEP(6)	SUBID	0.6368	0.09066	7.02	<.0001	Total Covariance for t-5
TOEP(7)	SUBID	0.6541	0.09179	7.13	<.0001	Total Covariance for t-6
Residual		0.8103	0.09014	8.99	<.0001	Total variance (equal across weeks)

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
926.0	7	940.0	940.1	948.0	959.7	966.7

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1.4884	0.07533	122	19.76	<.0001

The TOEP(*n*) model fits almost not worse than the n-order UN model, $-2\Delta LL(21) = 32.9$, $p = .047$ (and AIC and BIC agree).

Could this be our model? And does it need to have heterogeneous total variances across weeks?

Heterogeneous Toeplitz(*n*) R-only Model: Unequal total variances and 6 lagged total correlations

```
TITLE1 "SAS Heterogeneous Toeplitz (n=7 bands) R-only Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity = / SOLUTION DDFM=Satterthwaite;
       REPEATED week / TYPE=TOEPH(7) R RCORR SUBJECT=subid; RUN; TITLE1;
```

```
TITLE "SPSS Heterogeneous Toeplitz (n=7 bands) R-only Model".
MIXED severity BY subid week
   /METHOD = REML
   /PRINT = SOLUTION TESTCOV R
   /FIXED =
   /REPEATED = week | SUBJECT(subid) COVTYPE(TPH).
TITLE.
```

This one did estimate correctly in SPSS.

Heterogeneous variance models are not available in STATA as far as I could see.

SAS Output:

Estimated R Matrix for SUBID 100 → TOTAL variance and covariance

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.7346**	0.6916	0.6896	0.6409	0.6363	0.5985	0.6332
2	0.6916	**0.8077**	0.7512	0.7126	0.6866	0.6322	0.6485
3	0.6896	0.7512	**0.8665**	0.7667	0.7540	0.6738	0.6766
4	0.6409	0.7126	0.7667	**0.8414**	0.7718	0.7039	0.6861
5	0.6363	0.6866	0.7540	0.7718	**0.8781**	0.7470	0.7431
6	0.5985	0.6322	0.6738	0.7039	0.7470	**0.7882**	0.7314
7	0.6332	0.6485	0.6766	0.6861	0.7431	0.7314	**0.8416**

Estimated R Correlation Matrix for SUBID 100 → TOTAL correlation

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.8979	0.8644	0.8153	0.7923	0.7865	0.8053
2	**0.8979**	1.0000	0.8979	0.8644	0.8153	0.7923	0.7865
3	0.8644	**0.8979**	1.0000	0.8979	0.8644	0.8153	0.7923
4	0.8153	0.8644	**0.8979**	1.0000	0.8979	0.8644	0.8153
5	0.7923	0.8153	0.8644	**0.8979**	1.0000	0.8979	0.8644
6	0.7865	0.7923	0.8153	0.8644	**0.8979**	1.0000	0.8979
7	0.8053	0.7865	0.7923	0.8153	0.8644	**0.8979**	1.0000

Total *covariances are unequal* within a time lag band (because the total variances are allowed to differ over weeks), but total *correlations are equal* within a time lag band.

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
Var(1)	SUBID	0.7346	0.09397	7.82	<.0001	total variance at week1
Var(2)	SUBID	0.8077	0.1016	7.95	<.0001	total variance at week2
Var(3)	SUBID	0.8665	0.1129	7.68	<.0001	total variance at week3
Var(4)	SUBID	0.8414	0.1115	7.55	<.0001	total variance at week4
Var(5)	SUBID	0.8781	0.1162	7.56	<.0001	total variance at week5
Var(6)	SUBID	0.7882	0.1030	7.65	<.0001	total variance at week6
Var(7)	SUBID	0.8416	0.1092	7.70	<.0001	total variance at week7
TOEPH(1)	SUBID	0.8979	0.01292	69.51	<.0001	total correlation for t-1
TOEPH(2)	SUBID	0.8644	0.01744	49.57	<.0001	total correlation for t-2
TOEPH(3)	SUBID	0.8153	0.02429	33.56	<.0001	total correlation for t-3
TOEPH(4)	SUBID	0.7923	0.02790	28.40	<.0001	total correlation for t-4
TOEPH(5)	SUBID	0.7865	0.03056	25.73	<.0001	total correlation for t-5
TOEPH(6)	SUBID	0.8053	0.03340	24.11	<.0001	total correlation for t-6

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
921.5	13	947.5	947.9	962.3	984.1	997.1

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1.5204	0.07417	124	20.50	<.0001

Relative to the homogeneous Toeplitz: $-2\Delta LL(6) = 4.5$, $p = .609$. Separate variances don't improve model fit (AIC and BIC agree). The TOEPH(*n*) model also fits significantly worse than the UN(*n*) model, $-2\Delta LL(15) = 28.4$, $p = .019$.

Another possibility: Just as we'll see in models for growth, models for within-person fluctuation can make use of a combination of the **G** matrix (for between-person random effects) and the **R** matrix (for within-person residuals) to recapture the total variance-covariance matrix. Adding a random intercept variance in **G** removes the primary source of variance and covariance from the **R** matrix, such that it will be easier to find a model for what is left in **R**. First, let's see the pattern of just the RESIDUAL variances and covariances…

<u>Random Intercept in G + UN(*n*−1) R Model</u> → Have to eliminate last covariance for the model to be identified because there is only one lag6 covariance (T1 with T7), so it is not separately estimable

```
TITLE1 "SAS Random Intercept + Unstructured(n-1) R Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity = / SOLUTION DDFM=Satterthwaite;
       RANDOM INTERCEPT / TYPE=UN V VCORR G SUBJECT=subid;
       REPEATED week / TYPE=UN(6) R RCORR SUBJECT=subid; RUN; TITLE1;
```

SAS now has a random statement that specifies the **G** matrix to have a random intercept variance. This model is currently not available in SPSS or STATA.

SAS Output:

Estimated R Matrix for SUBID 100 → WP residual variances and covariances

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.1148**	0.08620	0.04892	0.04793	-0.02044	-0.01461	
2	0.08620	**0.1938**	0.1100	0.1147	0.02429	0.02907	0.02126
3	0.04892	0.1100	**0.2018**	0.1170	0.01962	0.006156	-0.01561
4	0.04793	0.1147	0.1170	**0.2038**	0.09643	0.08783	0.04774
5	-0.02044	0.02429	0.01962	0.09643	**0.1957**	0.09523	0.09853
6	-0.01461	0.02907	0.006156	0.08783	0.09523	**0.1583**	0.1024
7		0.02126	-0.01561	0.04774	0.09853	0.1024	**0.2136**

Estimated R Correlation Matrix for SUBID 100 → WP residual correlations

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.5778	0.3213	0.3133	-0.1364	-0.1084	
2	0.5778	1.0000	0.5563	0.5773	0.1247	0.1660	0.1045
3	0.3213	0.5563	1.0000	0.5771	0.09875	0.03445	-0.07519
4	0.3133	0.5773	0.5771	1.0000	0.4830	0.4891	0.2288
5	-0.1364	0.1247	0.09875	0.4830	1.0000	0.5412	0.4820
6	-0.1084	0.1660	0.03445	0.4891	0.5412	1.0000	0.5571
7		0.1045	-0.07519	0.2288	0.4820	0.5571	1.0000

Estimated G Matrix → BP random intercept variance

Row	Effect	subject id	Col1
1	Intercept	100	0.6328

This random intercept variance exactly matches the last lag covariance (7,1) from the n-order UN **R**-only model.

Estimated V Matrix for SUBID 100 → TOTAL variance and covariance (matches previous Unstructured R-only)

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	0.7477	0.7190	0.6817	0.6807	0.6124	0.6182	0.6328
2	0.7190	0.8266	0.7428	0.7475	0.6571	0.6619	0.6541
3	0.6817	0.7428	0.8346	0.7498	0.6524	0.6390	0.6172
4	0.6807	0.7475	0.7498	0.8366	0.7292	0.7206	0.6805
5	0.6124	0.6571	0.6524	0.7292	0.8285	0.7280	0.7313
6	0.6182	0.6619	0.6390	0.7206	0.7280	0.7911	0.7352
7	0.6328	0.6541	0.6172	0.6805	0.7313	0.7352	0.8464

Estimated V Correlation Matrix for SUBID 100 → TOTAL correlation (matches previous Unstructured RCORR)

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.9146	0.8630	0.8608	0.7781	0.8038	0.7955
2	0.9146	1.0000	0.8943	0.8989	0.7940	0.8185	0.7820
3	0.8630	0.8943	1.0000	0.8974	0.7846	0.7864	0.7343
4	0.8608	0.8989	0.8974	1.0000	0.8760	0.8859	0.8088

5	0.7781	0.7940	0.7846	0.8760	1.0000	0.8993	0.8734
6	0.8038	0.8185	0.7864	0.8859	0.8993	1.0000	0.8985
7	0.7955	0.7820	0.7343	0.8088	0.8734	0.8985	1.0000

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
893.1	28	949.1	951.3	981.2	1028.1	1056.1

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1.5178	0.07429	122	20.43	<.0001

The covariances/correlations in R appear to be minimal after removing the contribution of the random intercept variance, such that R may be able to have a simpler structure yet still fit well.

Just as we did with the **R**-only models, we can try a model with constant residual variance (e-only) in the **R** matrix as our most simple baseline for what should be in **R**. Accordingly, our output now includes the **V** and **VCORR** matrices, which put **G** and **R** (or **GCORR** and **RCORR**) back together again. Thus, the structure we end up with in V after combining the random intercept variance and residual variance is compound symmetry.

Random Intercept in G + Diagonal R Model → Same as compound symmetry R-only model

```
TITLE1 "SAS Random Intercept + Diagonal R Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity = / SOLUTION DDFM=Satterthwaite;
       RANDOM INTERCEPT / TYPE=UN V VCORR G SUBJECT=subid;
       REPEATED week / TYPE=VC R RCORR SUBJECT=subid; RUN; TITLE1;

TITLE "SPSS Random Intercept + Diagonal R Model".
MIXED severity BY subid week
   /METHOD = REML
   /PRINT = SOLUTION TESTCOV R
   /FIXED =
   /RANDOM = INTERCEPT | SUBJECT(subid) COVTYPE(UN).
   /REPEATED = week | SUBJECT(subid) COVTYPE(ID).
TITLE.

* STATA Random Intercept + Diagonal R Model
xtmixed severity ,  || subid:  , ///
       variance reml residuals(independent,t(Week)),
       estat ic, n(124),   // get AIC and BIC
```

SPSS also has a random statement with which to specify the **G** matrix to have a random intercept variance.

In STATA, we remove the "noconstant" option in the random part to include a random intercept variance in the **G** matrix.

SAS Output:

Estimated R Matrix for SUBID 100

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	0.1306						
2		0.1306					
3			0.1306				
4				0.1306			
5					0.1306		
6						0.1306	
7							0.1306

Estimated R Correlation Matrix for SUBID 100

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000						
2		1.0000					
3			1.0000				
4				1.0000			
5					1.0000		
6						1.0000	
7							1.0000

Estimated G Matrix

Row	Effect	subject id	Col1
1	Intercept	100	0.6821

Estimated V Matrix for SUBID 100

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	0.8127	0.6821	0.6821	0.6821	0.6821	0.6821	0.6821
2	0.6821	0.8127	0.6821	0.6821	0.6821	0.6821	0.6821
3	0.6821	0.6821	0.8127	0.6821	0.6821	0.6821	0.6821
4	0.6821	0.6821	0.6821	0.8127	0.6821	0.6821	0.6821
5	0.6821	0.6821	0.6821	0.6821	0.8127	0.6821	0.6821
6	0.6821	0.6821	0.6821	0.6821	0.6821	0.8127	0.6821
7	0.6821	0.6821	0.6821	0.6821	0.6821	0.6821	0.8127

Estimated VCORR Matrix for SUBID 100

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.8393	0.8393	0.8393	0.8393	0.8393	0.8393
2	0.8393	1.0000	0.8393	0.8393	0.8393	0.8393	0.8393
3	0.8393	0.8393	1.0000	0.8393	0.8393	0.8393	0.8393
4	0.8393	0.8393	0.8393	1.0000	0.8393	0.8393	0.8393
5	0.8393	0.8393	0.8393	0.8393	1.0000	0.8393	0.8393
6	0.8393	0.8393	0.8393	0.8393	0.8393	1.0000	0.8393
7	0.8393	0.8393	0.8393	0.8393	0.8393	0.8393	1.0000

The intraclass correlation in VCORR can be computed as:

$$\text{ICC} = \frac{0.6820}{0.6820 + 0.1306} = .84$$

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
UN(1,1)	SUBID	0.6821	0.09020	7.56	<.0001
Week	SUBID	0.1306	0.007269	17.97	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
1049.7	2	1053.7	1053.7	1056.0	1059.4	1061.4

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1.4767	0.07548	123	19.56	<.0001

We'll try two types of residual covariances/correlations: AR1 and reduced Toeplitz (covariances only at lags 5, 4, 3, 2, and 1). No reduced Toeplitz models are available in SPSS; heterogeneous variance versions are not available in STATA.

Random Intercept in G + AR1 R Model: only residuals have AR1 correlation (equal residual variance)

```
TITLE1 "SAS Random Intercept + AR1 R Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity = / SOLUTION DDFM=Satterthwaite;
       RANDOM INTERCEPT / V VCORR G TYPE=UN SUBJECT=subid;
       REPEATED week / TYPE=AR(1) R RCORR SUBJECT=subid; RUN; TITLE1;

TITLE "SPSS Random Intercept + AR1 R Model".
MIXED severity BY subid week
   /METHOD = REML
   /PRINT = SOLUTION TESTCOV R
   /FIXED =
   /RANDOM = INTERCEPT | SUBJECT(subid) COVTYPE(UN).
   /REPEATED = week | SUBJECT(subid) COVTYPE(AR1).
TITLE.

* STATA Random Intercept + AR1 R Model
xtmixed severity ,  || subid:  , ///
       variance reml residuals(ar1,t(Week)),
       estat ic, n(124),   // get AIC and BIC
```

SAS Output:

Estimated R Matrix for SUBID 100 → WP residual variance & covariance

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.1731**	0.08887	0.04564	0.02343	0.01203	0.006180	0.003173
2	0.08887	**0.1731**	0.08887	0.04564	0.02343	0.01203	0.006180
3	0.04564	0.08887	**0.1731**	0.08887	0.04564	0.02343	0.01203
4	0.02343	0.04564	0.08887	**0.1731**	0.08887	0.04564	0.02343
5	0.01203	0.02343	0.04564	0.08887	**0.1731**	0.08887	0.04564
6	0.006180	0.01203	0.02343	0.04564	0.08887	**0.1731**	0.08887
7	0.003173	0.006180	0.01203	0.02343	0.04564	0.08887	**0.1731**

Estimated R Correlation Matrix for SUBID 100 → WP residual correlation

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.5135	0.2637	0.1354	0.06954	0.03571	0.01834
2	**0.5135**	1.0000	0.5135	0.2637	0.1354	0.06954	0.03571
3	**0.2637**	**0.5135**	1.0000	0.5135	0.2637	0.1354	0.06954
4	**0.1354**	**0.2637**	**0.5135**	1.0000	0.5135	0.2637	0.1354
5	**0.06954**	**0.1354**	**0.2637**	**0.5135**	1.0000	0.5135	0.2637
6	**0.03571**	**0.06954**	**0.1354**	**0.2637**	**0.5135**	1.0000	0.5135
7	**0.01834**	**0.03571**	**0.06954**	**0.1354**	**0.2637**	**0.5135**	1.0000

Estimated G Matrix → BP random intercept variance

Row	Effect	subject id	Col1
1	Intercept	100	0.6350

Estimated V Matrix for SUBID 100 → TOTAL variance and covariance put back together

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.8080**	0.7238	0.6806	0.6584	0.6470	0.6411	0.6381
2	0.7238	**0.8080**	0.7238	0.6806	0.6584	0.6470	0.6411
3	0.6806	0.7238	**0.8080**	0.7238	0.6806	0.6584	0.6470
4	0.6584	0.6806	0.7238	**0.8080**	0.7238	0.6806	0.6584
5	0.6470	0.6584	0.6806	0.7238	**0.8080**	0.7238	0.6806
6	0.6411	0.6470	0.6584	0.6806	0.7238	**0.8080**	0.7238
7	0.6381	0.6411	0.6470	0.6584	0.6806	0.7238	**0.8080**

Estimated V Correlation Matrix for SUBID 100 → TOTAL correlation put back together

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7

1	1.0000	0.8958	0.8423	0.8148	0.8007	0.7935	0.7898
2	**0.8958**	1.0000	0.8958	0.8423	0.8148	0.8007	0.7935
3	**0.8423**	**0.8958**	1.0000	0.8958	0.8423	0.8148	0.8007
4	**0.8148**	**0.8423**	**0.8958**	1.0000	0.8958	0.8423	0.8148
5	**0.8007**	**0.8148**	**0.8423**	**0.8958**	1.0000	0.8958	0.8423
6	**0.7935**	**0.8007**	**0.8148**	**0.8423**	**0.8958**	1.0000	0.8958
7	**0.7898**	**0.7935**	**0.8007**	**0.8148**	**0.8423**	**0.8958**	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
UN(1,1)	SUBID	0.6350	0.09026	7.04	<.0001	Random Intercept variance (Var of U_0's)
AR(1)	SUBID	0.5135	0.05179	9.92	<.0001	The auto-regressive correlation in R
Residual		0.1731	0.01777	9.74	<.0001	Residual e variance (equal across weeks)

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
943.8	3	949.8	949.9	953.3	958.3	961.3

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1.4885	0.07515	122	19.81	<.0001

The AIC doesn't like this one better than our best model **R**-only so far (homogeneous Toeplitz), but the BIC could go either way. Although adding the random intercept variance is a significant improvement over the **R**-only AR1, but this still fits worse than UN(*n*-1).

Random Intercept in G with Toeplitz(6) in R Model (allows 5 lagged covariances; sets the last one to 0)

```
TITLE1 "SAS Random Intercept + Lag-5 Toeplitz R Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity = / SOLUTION DDFM=Satterthwaite;
       RANDOM INTERCEPT / V VCORR G TYPE=UN SUBJECT=subid;
       REPEATED week / TYPE=TOEP(6) R RCORR SUBJECT=subid; RUN; TITLE1;

* STATA Random Intercept + 5-lag Toeplitz R Model
xtmixed severity ,  || subid:  , ///
       variance reml residuals(toeplitz5,t(Week)),
       estat ic, n(124),   // get AIC and BIC
```

SAS Output:

Estimated R Matrix for SUBID 100 → WP residual variance & covariance

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.1562**	0.07152	0.04321	0.003458	-0.01479	-0.01730	
2	0.07152	**0.1562**	0.07152	0.04321	0.003458	-0.01479	-0.01730
3	0.04321	0.07152	**0.1562**	0.07152	0.04321	0.003458	-0.01479
4	0.003458	0.04321	0.07152	**0.1562**	0.07152	0.04321	0.003458
5	-0.01479	0.003458	0.04321	0.07152	**0.1562**	0.07152	0.04321
6	-0.01730	-0.01479	0.003458	0.04321	0.07152	**0.1562**	0.07152
7		-0.01730	-0.01479	0.003458	0.04321	0.07152	**0.1562**

Estimated R Correlation Matrix for SUBID 100 → WP residual correlation

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.4579	0.2766	0.02214	-0.09467	-0.1108	
2	0.4579	1.0000	0.4579	0.2766	0.02214	-0.09467	-0.1108
3	0.2766	0.4579	1.0000	0.4579	0.2766	0.02214	-0.09467
4	0.02214	0.2766	0.4579	1.0000	0.4579	0.2766	0.02214

5	-0.09467	0.02214	0.2766	0.4579	1.0000	0.4579	0.2766
6	-0.1108	-0.09467	0.02214	0.2766	0.4579	1.0000	0.4579
7		-0.1108	-0.09467	0.02214	0.2766	0.4579	1.0000

Estimated G Matrix → BP random intercept variance

Row	Effect	subject id	Col1
1	Intercept	100	0.6541

Estimated V Matrix for SUBID 100 → TOTAL variance and covariance put back together

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	0.8104	0.7257	0.6974	0.6576	0.6394	0.6368	0.6541
2	0.7257	0.8104	0.7257	0.6974	0.6576	0.6394	0.6368
3	0.6974	0.7257	0.8104	0.7257	0.6974	0.6576	0.6394
4	0.6576	0.6974	0.7257	0.8104	0.7257	0.6974	0.6576
5	0.6394	0.6576	0.6974	0.7257	0.8104	0.7257	0.6974
6	0.6368	0.6394	0.6576	0.6974	0.7257	0.8104	0.7257
7	0.6541	0.6368	0.6394	0.6576	0.6974	0.7257	0.8104

Estimated V Correlation Matrix for SUBID 100 → TOTAL correlation put back together

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.8955	0.8606	0.8115	0.7890	0.7859	0.8072
2	0.8955	1.0000	0.8955	0.8606	0.8115	0.7890	0.7859
3	0.8606	0.8955	1.0000	0.8955	0.8606	0.8115	0.7890
4	0.8115	0.8606	0.8955	1.0000	0.8955	0.8606	0.8115
5	0.7890	0.8115	0.8606	0.8955	1.0000	0.8955	0.8606
6	0.7859	0.7890	0.8115	0.8606	0.8955	1.0000	0.8955
7	0.8072	0.7859	0.7890	0.8115	0.8606	0.8955	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
UN(1,1)	SUBID	0.6541	0.09179	7.13	<.0001	Random Intercept U_0 Variance
TOEP(2)	SUBID	0.07152	0.02189	3.27	0.0011	Residual covariance for t-1
TOEP(3)	SUBID	0.04321	0.02130	2.03	0.0425	Residual covariance for t-2
TOEP(4)	SUBID	0.003458	0.02096	0.16	0.8690	Residual covariance for t-3
TOEP(5)	SUBID	-0.01479	0.01860	-0.80	0.4266	Residual covariance for t-4
TOEP(6)	SUBID	-0.01730	0.01487	-1.16	0.2444	Residual covariance for t-5
Residual		0.1562	0.02178	7.17	<.0001	Residual e Variance (equal across weeks)

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
926.0	7	940.0	940.1	948.0	959.7	966.7

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1.4884	0.07533	122	19.76	<.0001

This RI + TOEP(n−1) 5-lag model is equivalent to the Toeplitz(n) 6-lag model without the random intercept, as shown by the fit statistics. But this RI+5lag model has an interpretational advantage: rather than testing whether the total lagged covariance (in **V**) is different than 0, it tests whether the residual lagged covariance (in **R**) is different from 0 *after taking out individual differences in the intercept*. It looks like only some of the lagged covariances are significant, so we can probably simplify the model without hurting fit.

Summary of sequential models taking out higher-lag covariances:

TOEP(5) → 4 lags only: REML −2LL = 927.2 AIC = 939.2 BIC = 956.2
TOEP(4) → 3 lags only: REML −2LL = 927.3 AIC = 937.3 BIC = 951.4 ← Best so far
TOEP(3) → 2 lags only: REML −2LL = 933.6 AIC = 941.6 BIC = 952.8

Random Intercept in G with Toeplitz(3) in R Model (allows 3 lagged covariances; sets the others to 0)

```
TITLE1 "SAS Random Intercept + 3-Lag Toeplitz R Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity = / SOLUTION DDFM=Satterthwaite;
       RANDOM INTERCEPT / V VCORR G TYPE=UN SUBJECT=subid;
       REPEATED week / TYPE=TOEP(4) R RCORR SUBJECT=subid; RUN; TITLE1;

* STATA Random Intercept + 3-lag Toeplitz R Model
xtmixed severity ,  || subid:  , ///
       variance reml residuals(toeplitz3,t(Week)),
       estat ic, n(124),   // get AIC and BIC
```

SAS Output:

Estimated R Matrix for SUBID 100 → WP residual variance & covariance

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.1702**	0.08576	0.05791	0.01872			
2	0.08576	**0.1702**	0.08576	0.05791	0.01872		
3	0.05791	0.08576	**0.1702**	0.08576	0.05791	0.01872	
4	0.01872	0.05791	0.08576	**0.1702**	0.08576	0.05791	0.01872
5		0.01872	0.05791	0.08576	**0.1702**	0.08576	0.05791
6			0.01872	0.05791	0.08576	**0.1702**	0.08576
7				0.01872	0.05791	0.08576	**0.1702**

Estimated R Correlation Matrix for SUBID 100 → WP residual correlation

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.5038	0.3402	0.1100			
2	0.5038	1.0000	0.5038	0.3402	0.1100		
3	0.3402	0.5038	1.0000	0.5038	0.3402	0.1100	
4	0.1100	0.3402	0.5038	1.0000	0.5038	0.3402	0.1100
5		0.1100	0.3402	0.5038	1.0000	0.5038	0.3402
6			0.1100	0.3402	0.5038	1.0000	0.5038
7				0.1100	0.3402	0.5038	1.0000

Estimated G Matrix → BP random intercept variance

Row	Effect	subject id	Col1
1	Intercept	100	0.6395

Estimated V Matrix for SUBID 100

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	0.8097	0.7252	0.6974	0.6582	0.6395	0.6395	0.6395
2	0.7252	0.8097	0.7252	0.6974	0.6582	0.6395	0.6395
3	0.6974	0.7252	0.8097	0.7252	0.6974	0.6582	0.6395
4	0.6582	0.6974	0.7252	0.8097	0.7252	0.6974	0.6582
5	0.6395	0.6582	0.6974	0.7252	0.8097	0.7252	0.6974
6	0.6395	0.6395	0.6582	0.6974	0.7252	0.8097	0.7252
7	0.6395	0.6395	0.6395	0.6582	0.6974	0.7252	0.8097

Estimated V Correlation Matrix for SUBID 100

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.8957	0.8613	0.8129	0.7898	0.7898	0.7898
2	0.8957	1.0000	0.8957	0.8613	0.8129	0.7898	0.7898
3	0.8613	0.8957	1.0000	0.8957	0.8613	0.8129	0.7898
4	0.8129	0.8613	0.8957	1.0000	0.8957	0.8613	0.8129
5	0.7898	0.8129	0.8613	0.8957	1.0000	0.8957	0.8613
6	0.7898	0.7898	0.8129	0.8613	0.8957	1.0000	0.8957
7	0.7898	0.7898	0.7898	0.8129	0.8613	0.8957	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
UN(1,1)	SUBID	0.6395	0.09004	7.10	<.0001	Random Intercept U Variance
TOEP(2)	SUBID	0.08576	0.01267	6.77	<.0001	Residual covariance for t-1
TOEP(3)	SUBID	0.05791	0.009852	5.88	<.0001	Residual covariance for t-2
TOEP(4)	SUBID	0.01872	0.007469	2.51	0.0122	Residual covariance for t-3
Residual		0.1702	0.01401	12.15	<.0001	Residual e variance (equal across weeks)

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
927.3	5	937.3	937.4	943.1	951.4	956.4

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1.4899	0.07528	122	19.79	<.0001

One last thing to check: do the residual variances in **R** need to be heterogeneous?

Random Intercept in G with Heterogeneous Toeplitz(3) in R Model (allows 3 lagged covariances; sets the others to 0, adds unequal residual variances across weeks)

```
TITLE1 "SAS Random Intercept + 3-Lag Heterogeneous Toeplitz R Model";
PROC MIXED DATA=example2 COVTEST NOCLPRINT NOITPRINT IC METHOD=REML;
       CLASS subid week;
       MODEL severity = / SOLUTION DDFM=Satterthwaite;
       RANDOM INTERCEPT / V VCORR G TYPE=UN SUBJECT=subid;
       REPEATED week / TYPE=TOEPH(4) R RCORR SUBJECT=subid; RUN;
```

SAS Output:

Estimated R Matrix for SUBID 100 → WP residual variance & covariance

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.1261**	0.06946	0.06022	0.01485			
2	0.06946	**0.1462**	0.09301	0.05131	0.01985		
3	0.06022	0.09301	**0.2261**	0.09152	0.07918	0.02044	
4	0.01485	0.05131	0.09152	**0.1416**	0.08986	0.05187	0.01808
5		0.01985	0.07918	0.08986	**0.2180**	0.09231	0.07197
6			0.02044	0.05187	0.09231	**0.1494**	0.08546
7				0.01808	0.07197	0.08546	**0.1868**

Residual *covariances are unequal* within a time lag band (because the residual variances are allowed to differ over weeks), but the residual *correlations are equal* within a band.

Estimated R Correlation Matrix for SUBID 100 → WP residual correlation

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.5115	0.3566	0.1112			
2	0.5115	1.0000	0.5115	0.3566	0.1112		
3	0.3566	0.5115	1.0000	0.5115	0.3566	0.1112	
4	0.1112	0.3566	0.5115	1.0000	0.5115	0.3566	0.1112
5		0.1112	0.3566	0.5115	1.0000	0.5115	0.3566
6			0.1112	0.3566	0.5115	1.0000	0.5115
7				0.1112	0.3566	0.5115	1.0000

Estimated G Matrix → BP random intercept variance

Row	Effect	subject id	Col1
1	Intercept	100	0.6503

Estimated V Matrix for SUBID 100 → TOTAL variance and covariance put back together

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.7764**	0.7198	0.7105	0.6652	0.6503	0.6503	0.6503
2	0.7198	**0.7965**	0.7433	0.7016	0.6701	0.6503	0.6503
3	0.7105	0.7433	**0.8764**	0.7418	0.7295	0.6707	0.6503
4	0.6652	0.7016	0.7418	**0.7919**	0.7402	0.7022	0.6684
5	0.6503	0.6701	0.7295	0.7402	**0.8683**	0.7426	0.7223
6	0.6503	0.6503	0.6707	0.7022	0.7426	**0.7997**	0.7358
7	0.6503	0.6503	0.6503	0.6684	0.7223	0.7358	**0.8371**

Estimated V Correlation Matrix for SUBID 100 → TOTAL correlation put back together

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.9153	0.8614	0.8483	0.7920	0.8253	0.8066
2	0.9153	1.0000	0.8896	0.8834	0.8058	0.8148	0.7964
3	0.8614	0.8896	1.0000	0.8905	0.8362	0.8012	0.7592
4	0.8483	0.8834	0.8905	1.0000	0.8926	0.8824	0.8209
5	0.7920	0.8058	0.8362	0.8926	1.0000	0.8912	0.8472
6	0.8253	0.8148	0.8012	0.8824	0.8912	1.0000	0.8992
7	0.8066	0.7964	0.7592	0.8209	0.8472	0.8992	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
UN(1,1)	SUBID	0.6503	0.09069	7.17	<.0001	Random intercept U variance
Var(1)	SUBID	0.1261	0.02255	5.59	<.0001	Residual e Variance at week 1
Var(2)	SUBID	0.1462	0.02283	6.41	<.0001	Residual e Variance at week 2
Var(3)	SUBID	0.2261	0.03247	6.96	<.0001	Residual e Variance at week 3
Var(4)	SUBID	0.1416	0.02442	5.80	<.0001	Residual e Variance at week 4
Var(5)	SUBID	0.2180	0.03228	6.75	<.0001	Residual e Variance at week 5
Var(6)	SUBID	0.1494	0.02411	6.20	<.0001	Residual e Variance at week 6
Var(7)	SUBID	0.1868	0.02890	6.47	<.0001	Residual e Variance at week 7
TOEPH(1)	SUBID	0.5115	0.03899	13.12	<.0001	Residual correlation for t-1
TOEPH(2)	SUBID	0.3566	0.03764	9.48	<.0001	Residual correlation for t-2
TOEPH(3)	SUBID	0.1112	0.04115	2.70	0.0069	Residual correlation for t-3

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
910.0	11	932.0	932.3	944.6	963.0	974.0

The RI and homogeneous variance TOEP(4) 3-lag model is nested within the RI and heterogeneous TOEP(4) 3-lag model, so we can compare fit: $-2\Delta LL(6) = 17.3$, $p = .008$. Yes, separate variances improves model fit (AIC agrees, but BIC does not). One could go either way… in that case I'd probably re-check after adding predictors to see if it makes a difference. If not, it's parsimony vs. fit—your call.

Estimated R Matrix for SUBID 100 → TOTAL variance and covariance → ANSWER KEY

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	**0.7477**	0.7190	0.6817	0.6807	0.6124	0.6182	0.6328
2	0.7190	**0.8266**	0.7428	0.7475	0.6571	0.6619	0.6541
3	0.6817	0.7428	**0.8346**	0.7498	0.6524	0.6390	0.6172
4	0.6807	0.7475	0.7498	**0.8366**	0.7292	0.7206	0.6805
5	0.6124	0.6571	0.6524	0.7292	**0.8285**	0.7280	0.7313
6	0.6182	0.6619	0.6390	0.7206	0.7280	**0.7911**	0.7352
7	0.6328	0.6541	0.6172	0.6805	0.7313	0.7352	**0.8464**

Estimated R Correlation Matrix for SUBID 100 → TOTAL correlation → answer key

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.9146	0.8630	0.8608	0.7781	0.8038	0.7955
2	**0.9146**	1.0000	0.8943	0.8989	0.7940	0.8185	0.7820
3	**0.8630**	**0.8943**	1.0000	0.8974	0.7846	0.7864	0.7343
4	**0.8608**	**0.8989**	**0.8974**	1.0000	0.8760	0.8859	0.8088
5	**0.7781**	**0.7940**	**0.7846**	**0.8760**	1.0000	0.8993	0.8734
6	**0.8038**	**0.8185**	**0.7864**	**0.8859**	**0.8993**	1.0000	0.8985
7	**0.7955**	**0.7820**	**0.7343**	**0.8088**	**0.8734**	**0.8985**	1.0000