Other Kinds of Effects in General Linear Models (and Beyond)

- Today's Class:
  - Nonlinear effects: polynomial and exponential
  - Semi-continuous (piecewise) effects
  - Nested effects of mean differences or slopes

General "Linear" Model (Regression)

- Effects of continuous (quantitative) variables are usually described via a fixed linear slope relating X to Y
  - For example: $y_i = \beta_0 + \boldsymbol{\beta_1 X_i} + \cdots + e_i$
- However, that the X–Y relationship should be linear only is a testable hypothesis

- Many kinds of curvilinear functions—here are two examples:
  - **Polynomial**: effects of X^2 (predicts 1 bend) or X^3 (predicts 2 bends)
  - **Exponential**: effects of $LOG(X)$ → predicts 1 bend that asymptotes

- However, these terms are still part of a "linear" model
  - "Linear" means the fixed effects predict the conditional mean of the DV in a linear combination of (effect*predictor) + (effect*predictor)…

Quadratic Effects (e.g., of X=time)

<u>A Quadratic effect is a two-way interaction: time*time</u>

- Fixed quadratic time = "**<u>half</u> the rate of acceleration/deceleration**"
- So to interpret it as how the linear time effect changes per unit time, **you must multiply the quadratic coefficient by 2**

- If fixed linear time slope = 4 at time 0, with quadratic slope = 0.3?
  - Instantaneous linear rate of Δ at time 0 = 4.0, at time 1 = 4.6…

- The "twice" part comes from taking the derivatives of the function:

Intercept (Position) at Time T: $\hat{y}_T = 50.0 + 4.0T + 0.3T^2$

First Derivative (Velocity) at Time T: $\dfrac{d\hat{y}_T}{d(T)} = 4.0 + 0.6T$

Second Derivative (Acceleration) at Time T: $\dfrac{d^2\hat{y}_T}{d(T)} = 0.6$

Interpreting Quadratic Effects

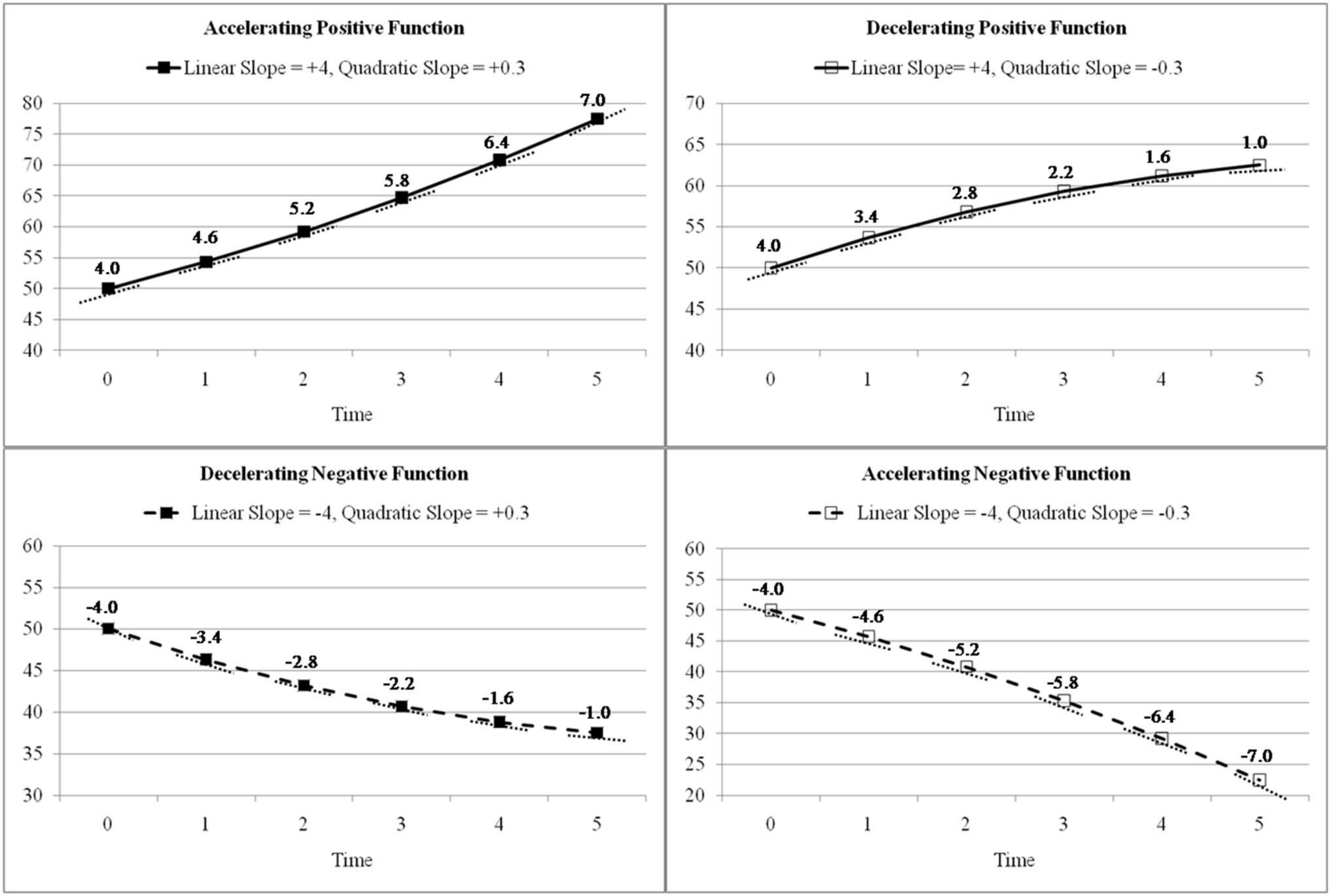
<u>A Quadratic time effect is a two-way interaction: time*time</u>

- Fixed quadratic = "**<u>half</u> the rate of acceleration/deceleration**"
- So to interpret it as how the linear time effect changes per unit time, **you must multiply the quadratic coefficient by 2**
- If fixed linear time slope = 4 at time 0, with quadratic slope = 0.3?
  - Instantaneous linear rate of Δ at time 0 = 4.0, at time 1 = 4.6…
- The "twice" part also comes from what you remember about the role of interactions with respect to their constituent main effects:

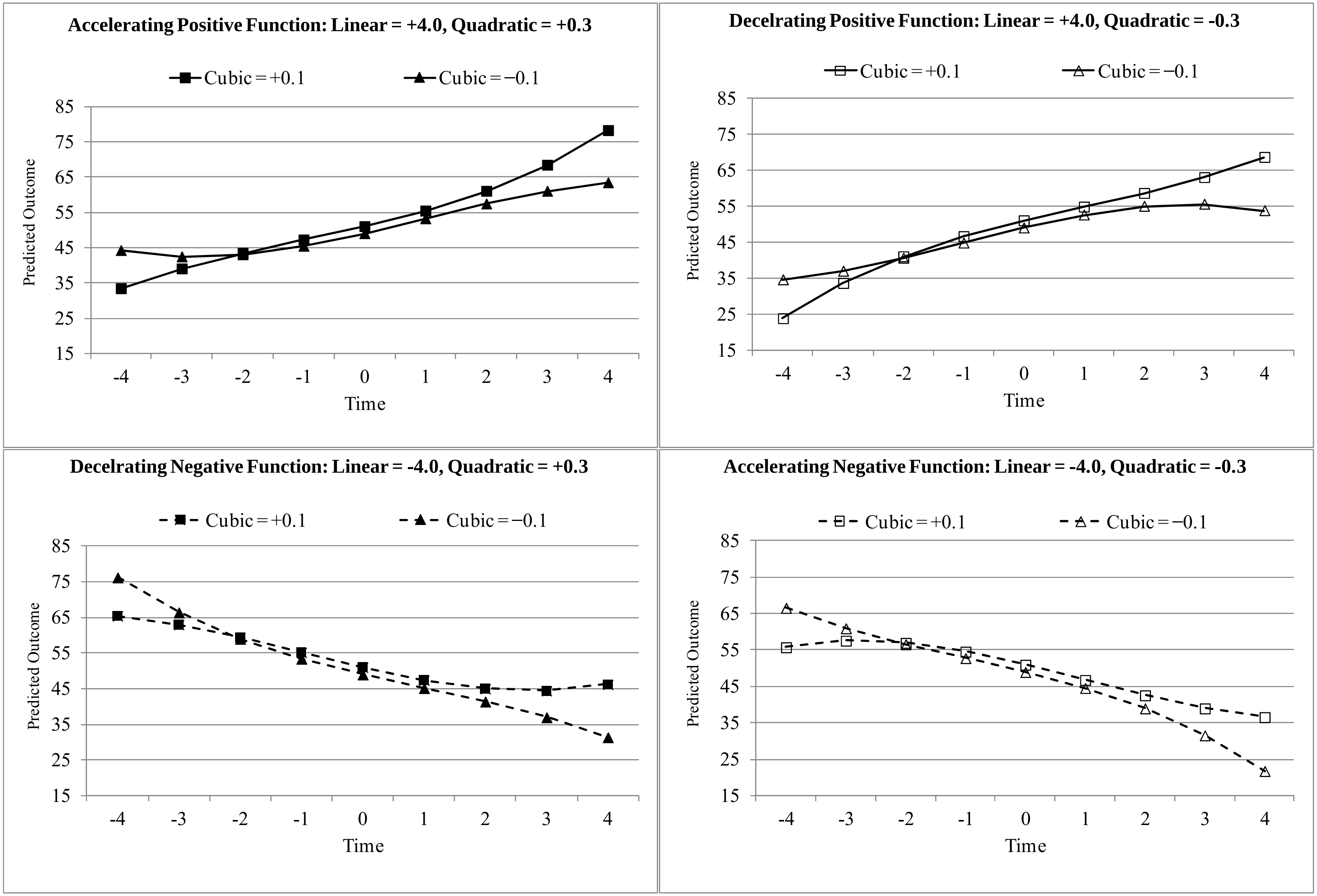
$$\hat{y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$$
$$\text{Effect of X} = \beta_1 + \beta_3 Z$$
$$\text{Effect of Z} = \beta_2 + \beta_3 X$$
$$\hat{y}_T = \beta_0 + \beta_1 \text{Time}_T + \underline{\qquad} + \beta_3 \text{Time}_T^2$$
$$\text{Effect of Time}_T = \beta_1 + 2\beta_3 \text{Time}_T$$

- Because time is interacting with itself, there is no second main effect in the model for the interaction to modify as usual. So the quadratic time effect gets applied <u>twice</u> to the <u>one</u> (main) linear effect of time.

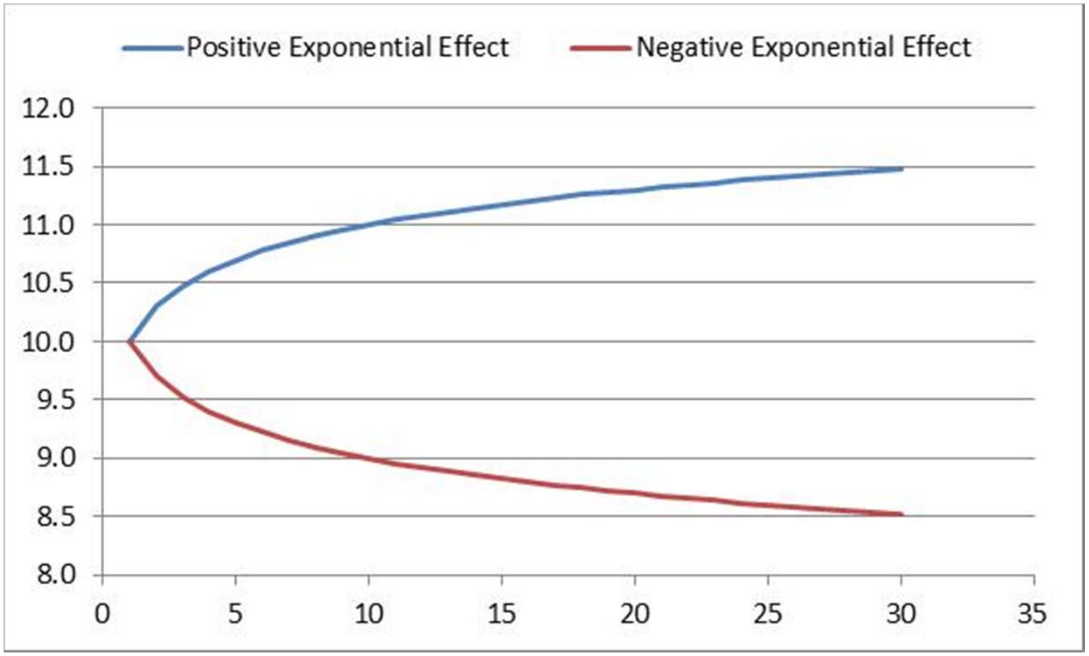
Examples of Quadratic Effects



Examples of Cubic Effects



Exponential Effects of LOG(X)



Create newX = LOG(X)

Example on left:
Positive slope = 1
Negative slope = −1

- Useful when the linear effect is expected to "shut off" as it moves towards extreme values (as opposed to change direction, which is what polynomial models predict instead)
- Example predictors: trial order, income, really skewed variables

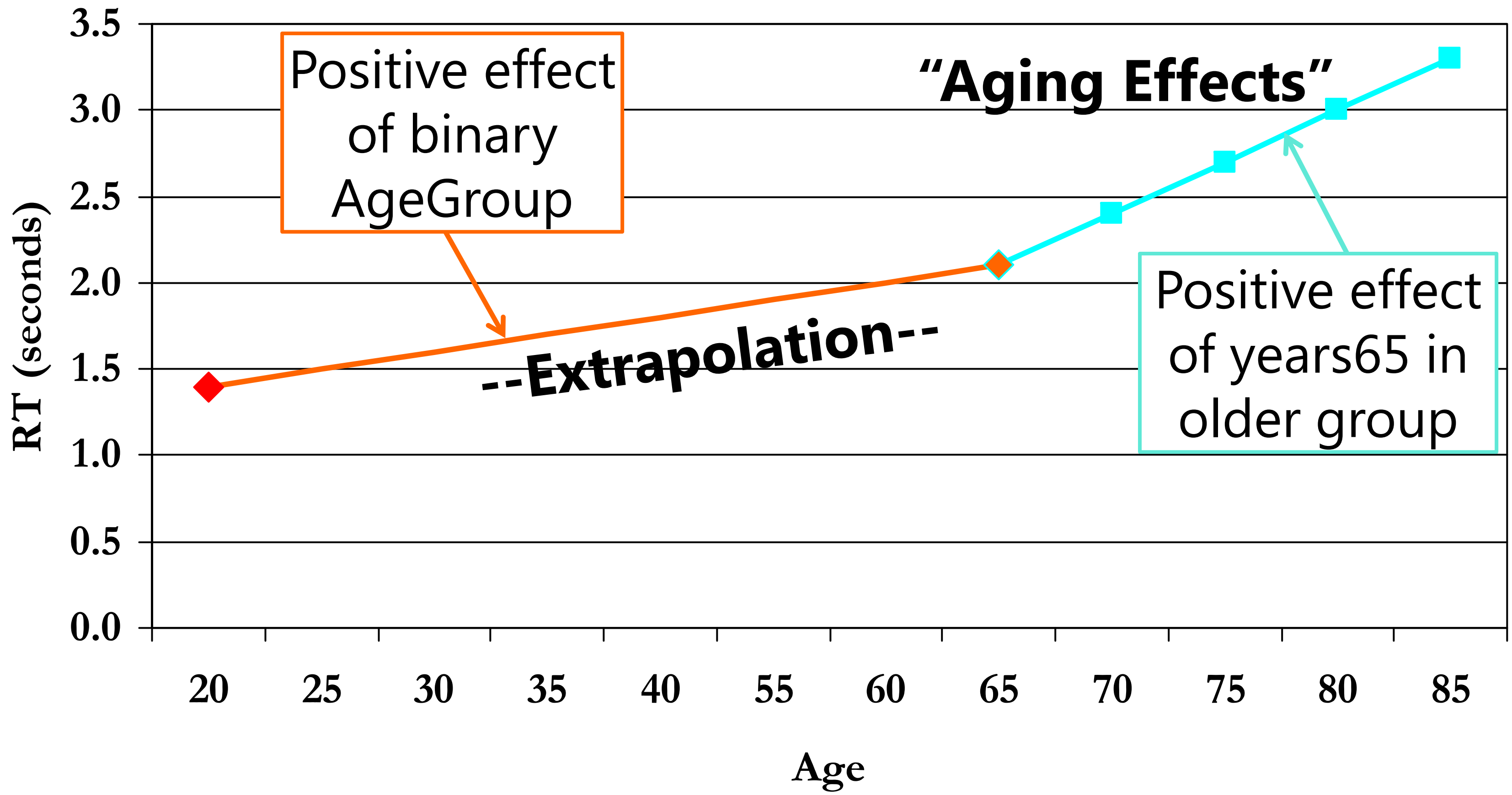
Semi-Continuous (Piecewise) Effects

- So far we've called predictors "continuous" or "categorical" but hybrid kinds are also possible
- **Semi-continuous** predictors will contain both qualitative and quantitative distinctions of "if" and "how much"
  - e.g., abuse severity, # chronic illnesses
- Example with "younger" and "older" adults: because age differences matter in the "older" group, 2 effects are needed:
  - "Age Group" → dummy code for difference of younger=0 and older=1
  - "Years over 65" → slope of age in the older group only

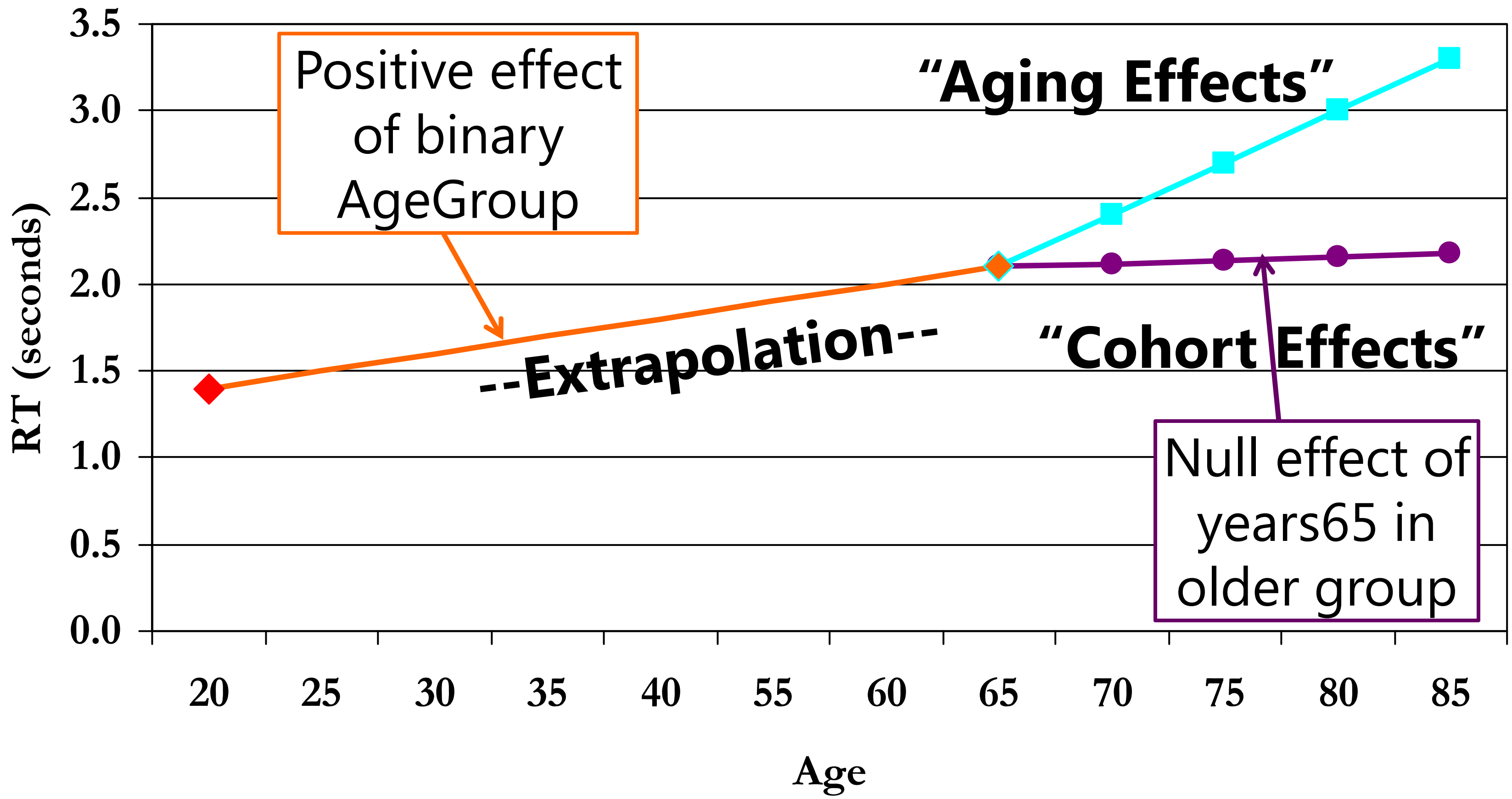
No age slope for younger: `IF AgeGroup=0 THEN years65=0;`

Create age slope for older: `IF AgeGroup=1 THEN years65=age-65;`

Piecewise (Semi-Continuous) Effects of Age on Response Time



Piecewise (Semi-Continuous) Effects of Age on Response Time



Categorical Predictors with Issues

- Experimental designs with fully crossed conditions lend themselves to analysis of variance-type models

- What happens when things go wrong? Two examples:
  - ANOVA with a hole in it
  - Predictors that don't apply to everyone

- These designs can be analyzed using nested effects
  - Software specifies these differently, so I'll show them via a common language of pseudo-interaction terms
  - "Interactions" act as switches instead to turn effects on/off

A Traditional View of ANOVA

ANOVAs are usually focused on F-tests for **marginal mean differences…**

$\mathbf{F(df=1)} \rightarrow \mathbf{a}\ v.\ \mathbf{b}$

$\mathbf{F(df=2)} \rightarrow \mathbf{1}\ v.\ \mathbf{2}.\ v.\ \mathbf{3}$

$\mathbf{F(df=2)} \rightarrow \mathbf{a}\ v.\ \mathbf{b} * \mathbf{1}\ v.\ \mathbf{2}.\ v.\ \mathbf{3}$

Is this **really** what you want to know?

Means	$\overline{\mathbf{1}}$	$\overline{\mathbf{2}}$	$\overline{\mathbf{3}}$
$\bar{\mathbf{a}}$	$\overline{\mathbf{a1}}$	$\overline{\mathbf{a2}}$	$\overline{\mathbf{a3}}$
$\bar{\mathbf{b}}$	$\overline{\mathbf{b1}}$	$\overline{\mathbf{b2}}$	$\overline{\mathbf{b3}}$

ANOVA as a Linear Model

$$
\begin{aligned}
\mathbf{y} = {} & \boxed{\boldsymbol{\beta}_0} + \boxed{\boldsymbol{\beta}_1(\mathbf{a1}\ v.\,\mathbf{b1})} \\
& + \boxed{\boldsymbol{\beta}_2(\mathbf{a1}\ v.\,\mathbf{a2}) + \boldsymbol{\beta}_3(a\mathbf{1}\ v.\,\mathbf{a3})} \\
& + \boxed{\begin{aligned} & \boldsymbol{\beta}_4(\mathbf{a1}\ v.\,\mathbf{b1})(\mathbf{a1}\ v.\,\mathbf{a2}) \\ + & \boldsymbol{\beta}_5(\mathbf{a1}\ v.\,\mathbf{b1})(\mathbf{a1}\ v.\,\mathbf{a3}) \end{aligned}} + \mathbf{e}
\end{aligned}
$$

The focus is now on **differences between *specific* conditions** as created by the **β** fixed effects.

Means	$\bar{\mathbf{1}}$	$\bar{\mathbf{2}}$	$\bar{\mathbf{3}}$
$\bar{\mathbf{a}}$	$\overline{\mathbf{a1}}$	$\overline{\mathbf{a2}}$	$\overline{\mathbf{a3}}$
$\bar{\mathbf{b}}$	$\overline{\mathbf{b1}}$	$\overline{\mathbf{b2}}$	$\overline{\mathbf{b3}}$

ANOVA as a Linear Model

- Software will find any **simple effects you ask for**
  - TEST in SPSS; ESTIMATE in SAS
  - LINCOM in STATA; NEW in Mplus

- Seeing research questions through linear models saves **nontraditional research designs**
  - Not fully crossed on purpose or by accident…

Means	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{a}$	β_0	$\beta_0 + \beta_2$	$\beta_0 + \beta_3$
$\bar{b}$	$\beta_0 + \beta_1$	$\beta_0 + \beta_1 + \beta_2 + \boxed{\beta_4}$	$\beta_0 + \beta_1 + \beta_3 + \boxed{\beta_5}$

A Nontraditional ANOVA Design

y = "ANOVA with a hole in it"

Problem: What can we do about Group 3?

Solution: Change the model to match the design

Means	Cohort 1	Cohort 2	Cohort 3
Control			
Treatment			

A Nontraditional ANOVA Design

$$y = \boxed{\beta_0} + \boxed{\beta_1(\text{t3 } v.\text{t1}) + \ \beta_2(\text{t2 } v.\text{t1})}$$
$$+ \boxed{\beta_3(\text{t1})(\text{t } v.\text{c})} + \boxed{\beta_4(\text{t2})(\text{t } v.\text{c})} + e$$

β_3 and β_4 are not interaction terms. Instead, they are *nested* effects.

You are allowed to use any *n* effects you want to represent the *n* means, even in fully crossed designs!

Means	Cohort 1	Cohort 2	Cohort 3
Control	$\beta_0 + \beta_1 + \boxed{\beta_3}$	$\beta_0 + \beta_2 + \boxed{\beta_4}$	
Treatment	$\beta_0 + \boxed{\beta_1}$	$\beta_0 + \boxed{\beta_2}$	$\boxed{\beta_0}$

A Nested-Effects General Linear Model

- Example: predicting outcomes by dementia type and timing in persons with OR without dementia
  - Type and timing do not apply to persons without dementia
  - So this requires the following new variables…

```
* Create a switch variable and nested type variable;
IF demtype="none" THEN DO; demYes=0; demAorV=  0; END;
IF demtype="AD"   THEN DO; demYes=1; demAorV=-.5; END;
IF demtype="VA"   THEN DO; demYes=1; demAorV= .5; END;
* Create a timing variable (0=5 years) when applicable;
IF demtype="none" THEN DO; demtime5=0;          END;
IF demtype="AD"   THEN DO; demtime5=demtime-5; END;
IF demtype="VA"   THEN DO; demtime5=demtime-5; END;
```

A Nested-Effects General Linear Model

- Example results would be interpreted as follows:

Fixed Effect	**Interpretation**
Intercept	Expected outcome for persons without dementia
demYes	Simple main effect for difference between persons without dementia or with dementia for 5 years (averaged across AD and VA dementia types)
demAorV	Simple main effect for difference between persons with AD or VA type dementia (for 5 years)
demYes* demtime5	Is NOT an interaction term: Slope for difference in outcome per year of dementia *only in persons with dementia* (averaged across AD and VA dementia types)
demAorV* demtime5	IS an interaction term: Difference in slope for effect of years between persons with AD or VA type

Wrapping Up…

- Effects of predictors can be specified in many ways
  - Continuous predictors: linear or nonlinear slopes
  - Categorical predictors: differences of whatever kind is of interest given your hypotheses and design (not just as in ANOVA formats)
  - Semi-continuous predictors: "if" and "how much" requires two+ effects → difference for "if"; (non)linear slope for "how much"

- Predictor effects can be specified for only the persons for whom they are relevant using nested effects instead
  - Omit main effects that do not apply to everyone, and replace with interactions that act as switches to let effects apply only to those persons for whom they are relevant
  - Make sure to fill in all irrelevant predictor values, too, otherwise the non-relevant people won't get included in the model