Piecewise Slopes Models for Practice Effects in Number Match 3 Response Times

The models for this example come from Hoffman (in preparation) chapter 6. We will be examining change in response time (RT) in milliseconds over six practice sessions to a measure of processing speed (as measured by the number match 3 test) in a sample of 101 older adults. Previously we used polynomial models to describe change in RT by session; now we use piecewise slope models instead.

SAS Syntax for Data Manipulation:

```
* Example for BALANCED DATA (everyone on same measurement schedule);
* Creating pieces with intercept at session 1, breakpoint at session 2;
* Making predictors for 2 Direct Slopes and Slope+Deviation Slope models;
DATA &datafile.; SET &datafile.;
            IF Session = 1 THEN DO; Slope16 = 0; Slope12 = 0; Slope26 = 0; END;
       ELSE IF Session = 2 THEN DO; Slope16 = 1; Slope12 = 1; Slope26 = 0; END;
       ELSE IF Session = 3 THEN DO; Slope16 = 2; Slope12 = 1; Slope26 = 1; END;
       ELSE IF Session = 4 THEN DO; Slope16 = 3; Slope12 = 1; Slope26 = 2; END;
       ELSE IF Session = 5 THEN DO; Slope16 = 4; Slope12 = 1; Slope26 = 3; END;
       ELSE IF Session = 6 THEN DO; Slope16 = 5; Slope12 = 1; Slope26 = 4; END;
       LABEL  Slope16 = "Slope16: Overall Slope from Session 1-6"
              Slope12 = "Slope12: Early Practice Slope (Session 1-2)"
              Slope26 = "Slope26: Later Practice Slope (Session 2-6)"; RUN;

* Example for UNBALANCED DATA (uses per-subject time variables instead);
DATA &datafile.; SET &datafile.;
     IF Session LE 1 THEN DO; Slope16=Time1-Time1; Slope12=Time1-Time1; Slope26=Time2-Time2; END;
ELSE IF Session LE 2 THEN DO; Slope16=Time2-Time1; Slope12=Time2-Time1; Slope26=Time2-Time2; END;
ELSE IF Session LE 3 THEN DO; Slope16=Time3-Time1; Slope12=Time2-Time1; Slope26=Time3-Time2; END;
ELSE IF Session LE 4 THEN DO; Slope16=Time4-Time1; Slope12=Time2-Time1; Slope26=Time4-Time2; END;
ELSE IF Session LE 5 THEN DO; Slope16=Time5-Time1; Slope12=Time2-Time1; Slope26=Time5-Time2; END;
ELSE IF Session LE 6 THEN DO; Slope16=Time6-Time1; Slope12=Time2-Time1; Slope26=Time6-Time2; END;
       LABEL   Slope16 = "Slope16: Overall Slope from Session 1-6"
               Slope12 = "Slope12: Early Practice Slope (Session 1-2)"
               Slope26 = "Slope26: Later Practice Slope (Session 2-6)"; RUN;
```

Model 0: Saturated Means, Unstructured Variance Model (the answer key best baseline model)

```
TITLE1 "Model 0: Saturated Means, Unstructured Variances Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
       CLASS ID session;
       MODEL nm3rt = session / SOLUTION DDFM=Satterthwaite;
       REPEATED session / R RCORR TYPE=UN SUBJECT=ID;
       LSMEANS session /; RUN;
```

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**301985**	235659	217994	202607	192154	195360
2	235659	**259150**	230217	213232	202092	193268
3	217994	230217	**233368**	205209	196919	188604
4	202607	213232	205209	**217544**	193676	185321
5	192154	202092	196919	193676	**212098**	187840
6	195360	193268	188604	185321	187840	**196733**

This **R matrix** UN structure lets all variances and covariances be what they want. THIS IS THE DATA we are trying to duplicate with our model for the variances.

Information Criteria

Neg2LogLike	**Parms**	AIC	AICC	HQIC	BIC	CAIC
8229.8	**21**	8271.8	8273.4	8294.0	8326.7	8347.7

Least Squares Means

Effect	Session (1-6)	Estimate	Standard Error	DF	t Value	Pr > |t|
Session	1	1961.89	54.6805	100	35.88	<.0001
Session	2	1815.17	50.6541	100	35.83	<.0001
Session	3	1750.03	48.0684	100	36.41	<.0001
Session	4	1717.80	46.4101	100	37.01	<.0001
Session	5	1707.18	45.8255	100	37.25	<.0001
Session	6	1672.14	44.1345	100	37.89	<.0001

The **saturated means model** lets all session means be what they want. THIS IS THE DATA we are trying to duplicate with our model for the means.

Model 4a: Fixed Slope12, Fixed Slope26, Random Intercept Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Slope12}_{ti}) + \beta_{2i}(\text{Slope26}_{ti}) + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10}$

Slope26: $\beta_{2i} = \gamma_{20}$

```
TITLE1 "Model 4a: Fixed Slope12, Fixed Slope26, Random Intercept Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = Slope12 Slope26 / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT / V VCORR TYPE=UN SUBJECT=ID;
  REPEATED session / R TYPE=VC SUBJECT=ID;
  ESTIMATE "Session 1 Predicted Mean" Intercept 1 Slope12 0 Slope26 0;
  ESTIMATE "Session 2 Predicted Mean" Intercept 1 Slope12 1 Slope26 0;
  ESTIMATE "Session 3 Predicted Mean" Intercept 1 Slope12 1 Slope26 1;
  ESTIMATE "Session 4 Predicted Mean" Intercept 1 Slope12 1 Slope26 2;
  ESTIMATE "Session 5 Predicted Mean" Intercept 1 Slope12 1 Slope26 3;
  ESTIMATE "Session 6 Predicted Mean" Intercept 1 Slope12 1 Slope26 4;
  ESTIMATE "Difference between slopes" Slope12 -1 Slope26 1;
RUN;
```

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9188.48345679	
1	1	**8382.68712287**	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	34098					
2		34098				
3			34098			
4				34098		
5					34098	
6						34098

This **R matrix** VC structure (equal variance over time, no covariance of any kind) will be used repeatedly as we add fixed and random piecewise slopes to the model.

Estimated G Matrix

Row	Effect	Person ID	Col1
1	Intercept	101	202683

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**236781**	202683	202683	202683	202683	202683
2	202683	**236781**	202683	202683	202683	202683
3	202683	202683	**236781**	202683	202683	202683
4	202683	202683	202683	**236781**	202683	202683
5	202683	202683	202683	202683	**236781**	202683
6	202683	202683	202683	202683	202683	**236781**

This random intercept model predicts a compound symmetry pattern for the **V** matrix (equal variance, equal covariance over time).

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8560	0.8560	0.8560	0.8560	0.8560
2	0.8560	1.0000	0.8560	0.8560	0.8560	0.8560
3	0.8560	0.8560	1.0000	0.8560	0.8560	0.8560
4	0.8560	0.8560	0.8560	1.0000	0.8560	0.8560
5	0.8560	0.8560	0.8560	0.8560	1.0000	0.8560
6	0.8560	0.8560	0.8560	0.8560	0.8560	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
UN(1,1)	ID	202683	29470	6.88	<.0001
Session	ID	34098	2150.11	15.86	<.0001

Fit Statistics

-2 Res Log Likelihood	8382.7
AIC (smaller is better)	8386.7
AICC (smaller is better)	8386.7
BIC (smaller is better)	8391.9

Are the fixed piecewise slopes significant? How do we know?

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	805.80	<.0001

Information Criteria

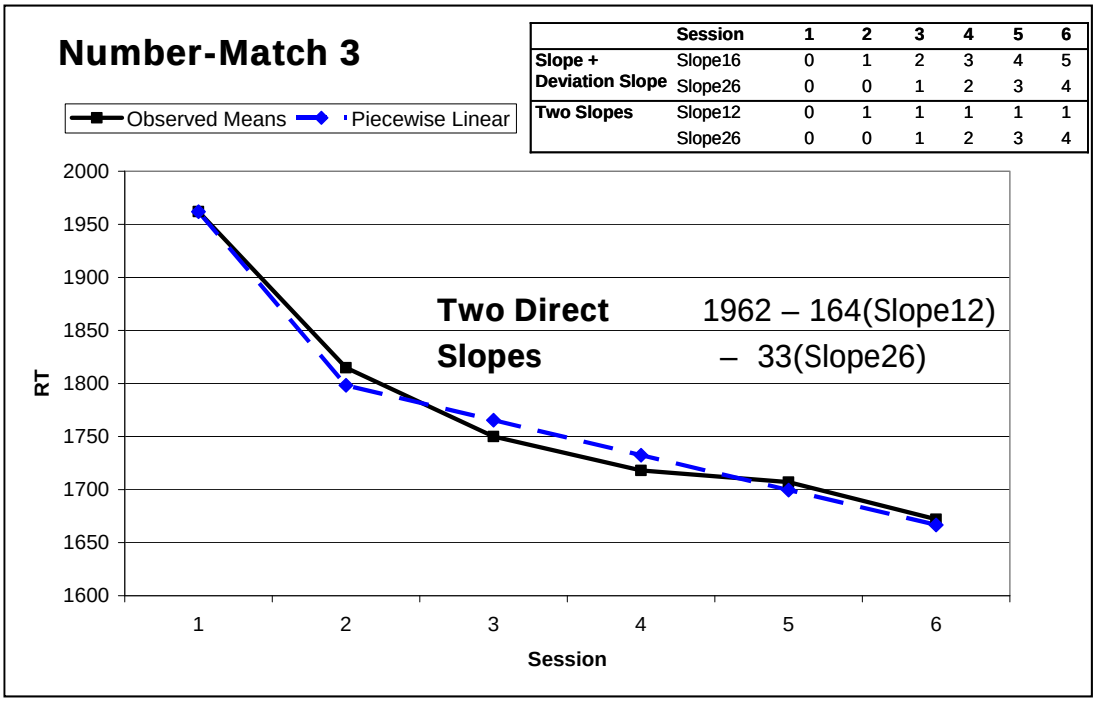
Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8382.7	2	8386.7	8386.7	8388.8	8391.9	8393.9

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1961.89	48.4187	129	40.52	<.0001
Slope12	-163.64	23.2415	503	-7.04	<.0001
Slope26	-32.8932	5.8104	503	-5.66	<.0001

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Session 1 Predicted Mean	1961.89	48.4187	129	40.52	<.0001
Session 2 Predicted Mean	1798.25	47.0035	115	38.26	<.0001
Session 3 Predicted Mean	1765.36	45.9134	104	38.45	<.0001
Session 4 Predicted Mean	1732.46	45.5443	101	38.04	<.0001
Session 5 Predicted Mean	1699.57	45.9134	104	37.02	<.0001
Session 6 Predicted Mean	1666.68	47.0035	115	35.46	<.0001
Difference between slopes	130.75	26.6265	503	4.91	<.0001



Model 4b: Random Slope12, Fixed Slope26 Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Slope12}_{ti}) + \beta_{2i}(\text{Slope26}_{ti}) + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20}$

```
TITLE1 "Model 4b: Random Slope12, Fixed Slope26 Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = Slope12 Slope26 / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT Slope12 / G GCORR V VCORR TYPE=UN SUBJECT=ID;
  REPEATED session / R TYPE=VC SUBJECT=ID;
  ESTIMATE "Session 1 Predicted Mean" Intercept 1 Slope12 0 Slope26 0;
  ESTIMATE "Session 2 Predicted Mean" Intercept 1 Slope12 1 Slope26 0;
  ESTIMATE "Session 3 Predicted Mean" Intercept 1 Slope12 1 Slope26 1;
  ESTIMATE "Session 4 Predicted Mean" Intercept 1 Slope12 1 Slope26 2;
  ESTIMATE "Session 5 Predicted Mean" Intercept 1 Slope12 1 Slope26 3;
  ESTIMATE "Session 6 Predicted Mean" Intercept 1 Slope12 1 Slope26 4;
  ESTIMATE "Difference between slopes" Slope12 -1 Slope26 1;
RUN;
```

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9188.48345679	
1	1	**8319.61326151**	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	24168					
2		24168				
3			24168			
4				24168		
5					24168	
6						24168

Estimated G Matrix

Row	Effect	Person ID	Col1	Col2
1	Intercept	101	277818	-69063
2	Slope12	101	-69063	59941

Estimated G Correlation Matrix

Row	Effect	Person ID	Col1	Col2
1	Intercept	101	1.0000	-0.5352
2	Slope12	101	-0.5352	1.0000

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**301985**	208755	208755	208755	208755	208755
2	208755	**223800**	199632	199632	199632	199632
3	208755	199632	**223800**	199632	199632	199632
4	208755	199632	199632	**223800**	199632	199632
5	208755	199632	199632	199632	**223800**	199632
6	208755	199632	199632	199632	199632	**223800**

How would we describe the pattern of variance and covariances in **V** now?

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8030	0.8030	0.8030	0.8030	0.8030
2	0.8030	1.0000	0.8920	0.8920	0.8920	0.8920
3	0.8030	0.8920	1.0000	0.8920	0.8920	0.8920
4	0.8030	0.8920	0.8920	1.0000	0.8920	0.8920
5	0.8030	0.8920	0.8920	0.8920	1.0000	0.8920
6	0.8030	0.8920	0.8920	0.8920	0.8920	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	277818	42741	6.50	<.0001
UN(2,1)	ID	-69063	18932	-3.65	0.0003
UN(2,2)	ID	59941	12743	4.70	<.0001
Session	ID	24168	1702.53	14.20	<.0001

Fit Statistics

-2 Res Log Likelihood	8319.6
AIC (smaller is better)	8327.6
AICC (smaller is better)	8327.7
BIC (smaller is better)	8338.1

Is the random slope12 significant? How do we know?

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
3	868.87	<.0001

Information Criteria

Neg2LogLike	**Parms**	AIC	AICC	HQIC	BIC	CAIC
8319.6	**4**	8327.6	8327.7	8331.8	8338.1	8342.1

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1961.89	54.6805	100	35.88	<.0001
Slope12	-163.64	31.2462	123	-5.24	<.0001
Slope26	-32.8932	4.8916	403	-6.72	<.0001

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Session 1 Predicted Mean	1961.89	54.6805	100	35.88	<.0001
Session 2 Predicted Mean	1798.25	46.0448	110	39.05	<.0001
Session 3 Predicted Mean	1765.36	45.2586	102	39.01	<.0001
Session 4 Predicted Mean	1732.46	44.9935	100	38.50	<.0001
Session 5 Predicted Mean	1699.57	45.2586	102	37.55	<.0001
Session 6 Predicted Mean	1666.68	46.0448	110	36.20	<.0001
Difference between slopes	130.75	33.1054	153	3.95	0.0001

What kind of individual differences in change does this model predict?

Model 4c: Random Slope12, Random Slope26 Model

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Slope12}_{ti}\right) + \beta_{2i}\left(\text{Slope26}_{ti}\right) + e_{ti}$$

$$\text{Level 2: Intercept: } \beta_{0i} = \gamma_{00} + U_{0i}$$

$$\text{Slope12: } \beta_{1i} = \gamma_{10} + U_{1i}$$

$$\text{Slope26: } \beta_{2i} = \gamma_{20} + U_{2i}$$

```
TITLE1 "Model 4c: Random Slope12, Random Slope26 Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = Slope12 Slope26 / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT Slope12 Slope26 / G GCORR V VCORR TYPE=UN SUBJECT=ID;
  REPEATED session / R TYPE=VC SUBJECT=ID;
  ESTIMATE "Session 1 Predicted Mean" Intercept 1 Slope12 0 Slope26 0;
  ESTIMATE "Session 2 Predicted Mean" Intercept 1 Slope12 1 Slope26 0;
  ESTIMATE "Session 3 Predicted Mean" Intercept 1 Slope12 1 Slope26 1;
  ESTIMATE "Session 4 Predicted Mean" Intercept 1 Slope12 1 Slope26 2;
  ESTIMATE "Session 5 Predicted Mean" Intercept 1 Slope12 1 Slope26 3;
  ESTIMATE "Session 6 Predicted Mean" Intercept 1 Slope12 1 Slope26 4;
  ESTIMATE "Difference between slopes" Slope12 -1 Slope26 1;
RUN;
```

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9188.48345679	
1	1	**8275.37431715**	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	17673					
2		17673				
3			17673			
4				17673		
5					17673	
6						17673

Estimated G Matrix

Row	Effect	Person ID	Col1	Col2	Col3
1	Intercept	101	284312	-54270	-10644
2	Slope12	101	-54270	63954	-1672.30
3	Slope26	101	-10644	-1672.30	2617.28

Estimated G Correlation Matrix

Row	Effect	Person ID	Col1	Col2	Col3
1	Intercept	101	1.0000	-0.4025	-0.3902
2	Slope12	101	-0.4025	1.0000	-0.1293
3	Slope26	101	-0.3902	-0.1293	1.0000

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**301985**	230042	219399	208755	198111	187467
2	230042	**257400**	227410	215094	202778	190462
3	219399	227410	**235385**	208013	198314	188615
4	208755	215094	208013	**218604**	193850	186768
5	198111	202778	198314	193850	**207059**	184921
6	187467	190462	188615	186768	184921	**200747**

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8251	0.8229	0.8125	0.7923	0.7614
2	0.8251	1.0000	0.9239	0.9068	0.8784	0.8379
3	0.8229	0.9239	1.0000	0.9170	0.8983	0.8677

4	0.8125	0.9068	0.9170	1.0000	0.9111	0.8916
5	0.7923	0.8784	0.8983	0.9111	1.0000	0.9070
6	0.7614	0.8379	0.8677	0.8916	0.9070	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	284312	42731	6.65	<.0001
UN(2,1)	ID	-54270	18230	-2.98	0.0029
UN(2,2)	ID	63954	13244	4.83	<.0001
UN(3,1)	ID	-10644	3791.26	-2.81	0.0050
UN(3,2)	ID	-1672.30	2097.03	-0.80	0.4252
UN(3,3)	ID	2617.28	636.48	4.11	<.0001
Session	ID	17673	1435.84	12.31	<.0001

Fit Statistics

-2 Res Log Likelihood	8275.4
AIC (smaller is better)	8289.4
AICC (smaller is better)	8289.6
BIC (smaller is better)	8307.7

Is the random slope26 significant? How do we know?

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
6	913.11	<.0001

Information Criteria

Neg2LogLike	**Parms**	AIC	AICC	HQIC	BIC	CAIC
8275.4	**7**	8289.4	8289.6	8296.8	8307.7	8314.7

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1961.89	54.6805	100	35.88	<.0001
Slope12	-163.64	30.2188	100	-5.42	<.0001
Slope26	-32.8932	6.5888	100	-4.99	<.0001

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Session 1 Predicted Mean	1961.89	54.6805	100	35.88	<.0001
Session 2 Predicted Mean	1798.25	49.7847	100	36.12	<.0001
Session 3 Predicted Mean	1765.36	46.9899	100	37.57	<.0001
Session 4 Predicted Mean	1732.46	44.9935	100	38.50	<.0001
Session 5 Predicted Mean	1699.57	43.9044	100	38.71	<.0001
Session 6 Predicted Mean	1666.68	43.7905	100	38.06	<.0001
Difference between slopes	130.75	32.5530	100	4.02	0.0001

What kind of individual differences in change does this model predict?

$$\text{Random Effect 95\% CI} = \text{fixed effect} \pm \left(1.96 * \sqrt{\text{Random Variance}}\right)$$

$$\text{Intercept 95\% CI} = \gamma_{00} \pm \left(1.96 * \sqrt{\tau^2_{U_0}}\right) \rightarrow \; 1{,}961.9 \pm \left(1.96 * \sqrt{284{,}312}\right) = 917 \text{ to } 3{,}007$$

$$\text{Slope12 95\% CI} = \gamma_{10} \pm \left(1.96 * \sqrt{\tau^2_{U_1}}\right) \rightarrow \; -163.6 \pm \left(1.96 * \sqrt{63{,}954}\right) = -659 \text{ to } 322$$

$$\text{Slope26 95\% CI} = \gamma_{20} \pm \left(1.96 * \sqrt{\tau^2_{U_2}}\right) \rightarrow \; -32.9 \pm \left(1.96 * \sqrt{2{,}617}\right) = -133 \text{ to } 67$$

So far we've examined one way to fit piecewise slopes models—direct slopes that represent the change during each time period. Let's now examine an alternative specification—slope + deviation slope, which can be useful in examining individual differences in differential change between time periods.

Model 5a: Fixed Slope, Fixed Deviation Slope, Random Intercept Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(Slope16_{ti}) + \beta_{2i}(Slope26_{ti}) + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10}$

Slope26: $\beta_{2i} = \gamma_{20}$

```
TITLE1 "Model 5a: Fixed Slope, Fixed Deviation Slope, Random Intercept Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = Slope16 Slope26 / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT / G V VCORR TYPE=UN SUBJECT=ID;
  REPEATED session / R TYPE=VC SUBJECT=ID;
  ESTIMATE "Session 1 Mean" Intercept 1 Slope16 0 Slope26 0;
  ESTIMATE "Session 2 Mean" Intercept 1 Slope16 1 Slope26 0;
  ESTIMATE "Session 3 Mean" Intercept 1 Slope16 2 Slope26 1;
  ESTIMATE "Session 4 Mean" Intercept 1 Slope16 3 Slope26 2;
  ESTIMATE "Session 5 Mean" Intercept 1 Slope16 4 Slope26 3;
  ESTIMATE "Session 6 Mean" Intercept 1 Slope16 5 Slope26 4;
  ESTIMATE "Slope from session 2-6" Slope16 1 Slope26 1;
RUN;
```

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9188.48345679	
1	1	**8382.68712287**	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	34098					
2		34098				
3			34098			
4				34098		
5					34098	
6						34098

Estimated G Matrix

Row	Effect	Person ID	Col1
1	Intercept	101	202683

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	236781	202683	202683	202683	202683	202683
2	202683	236781	202683	202683	202683	202683
3	202683	202683	236781	202683	202683	202683
4	202683	202683	202683	236781	202683	202683
5	202683	202683	202683	202683	236781	202683
6	202683	202683	202683	202683	202683	236781

This random intercept model predicts a compound symmetry pattern for the **V** matrix (equal variance, equal covariance over time).

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8560	0.8560	0.8560	0.8560	0.8560
2	0.8560	1.0000	0.8560	0.8560	0.8560	0.8560
3	0.8560	0.8560	1.0000	0.8560	0.8560	0.8560
4	0.8560	0.8560	0.8560	1.0000	0.8560	0.8560
5	0.8560	0.8560	0.8560	0.8560	1.0000	0.8560
6	0.8560	0.8560	0.8560	0.8560	0.8560	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
UN(1,1)	ID	202683	29470	6.88	<.0001
Session	ID	34098	2150.11	15.86	<.0001

Fit Statistics

-2 Res Log Likelihood	8382.7
AIC (smaller is better)	8386.7
AICC (smaller is better)	8386.7
BIC (smaller is better)	8391.9

Are the fixed piecewise slopes significant? How do we know?

Null Model Likelihood Ratio Test

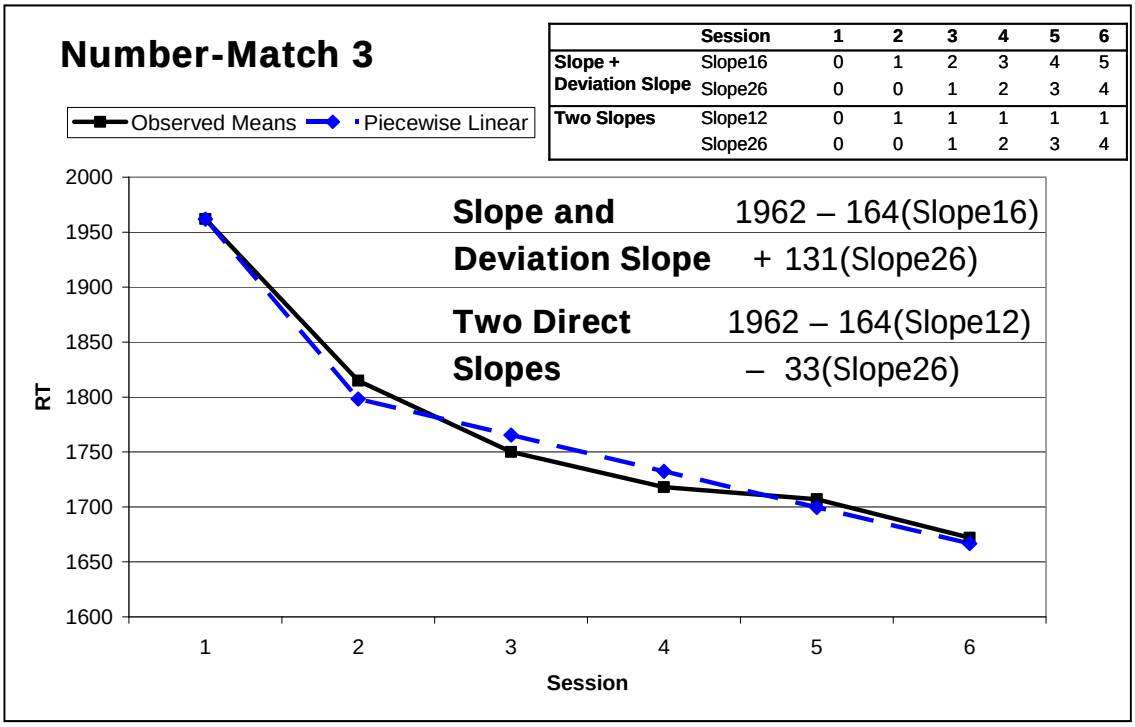
DF	Chi-Square	Pr > ChiSq
1	805.80	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1961.89	48.4187	129	40.52	<.0001
Slope16	-163.64	23.2415	503	-7.04	<.0001
Slope26	130.75	26.6265	503	4.91	<.0001

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Session 1 Mean	1961.89	48.4187	129	40.52	<.0001
Session 2 Mean	1798.25	47.0035	115	38.26	<.0001
Session 3 Mean	1765.36	45.9134	104	38.45	<.0001
Session 4 Mean	1732.46	45.5443	101	38.04	<.0001
Session 5 Mean	1699.57	45.9134	104	37.02	<.0001
Session 6 Mean	1666.68	47.0035	115	35.46	<.0001
Slope from session 2-6	-32.8932	5.8104	503	-5.66	<.0001



	Session	1	2	3	4	5	6
Slope + Deviation Slope	Slope16	0	1	2	3	4	5
	Slope26	0	0	1	2	3	4
Two Slopes	Slope12	0	1	1	1	1	1
	Slope26	0	0	1	2	3	4

Model 5b: Random Slope, Fixed Deviation Slope Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(Slope16_{ti}) + \beta_{2i}(Slope26_{ti}) + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20}$

```
TITLE1 "Model 5b: Random Slope, Fixed Deviation Slope Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = Slope16 Slope26 / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT Slope16 / G GCORR V VCORR TYPE=UN SUBJECT=ID;
  REPEATED session / R TYPE=VC SUBJECT=ID;
  ESTIMATE "Session 1 Mean" Intercept 1 Slope16 0 Slope26 0;
  ESTIMATE "Session 2 Mean" Intercept 1 Slope16 1 Slope26 0;
  ESTIMATE "Session 3 Mean" Intercept 1 Slope16 2 Slope26 1;
  ESTIMATE "Session 4 Mean" Intercept 1 Slope16 3 Slope26 2;
  ESTIMATE "Session 5 Mean" Intercept 1 Slope16 4 Slope26 3;
  ESTIMATE "Session 6 Mean" Intercept 1 Slope16 5 Slope26 4;
  ESTIMATE "Slope from session 2-6" Slope16 1 Slope26 1;
RUN;
```

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9188.48345679	
1	1	**8333.38121978**	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	25934					
2		25934				
3			25934			
4				25934		
5					25934	
6						25934

Estimated G Matrix

Row	Effect	Person ID	Col1	Col2
1	Intercept	101	254290	-12982
2	Slope16	101	-12982	2346.46

Estimated G Correlation Matrix

Row	Effect	Person ID	Col1	Col2
1	Intercept	101	1.0000	-0.5315
2	Slope16	101	-0.5315	1.0000

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**280225**	241308	228325	215343	202361	189378
2	241308	**256606**	220036	209400	198764	188128
3	228325	220036	**237681**	203457	195168	186878
4	215343	209400	203457	**223449**	191571	185628
5	202361	198764	195168	191571	**213909**	184378
6	189378	188128	186878	185628	184378	**209063**

This random slope model predicts the same kind of **V** matrix as would a random linear time model —on the random side, that's what this is!

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8999	0.8847	0.8606	0.8265	0.7824
2	0.8999	1.0000	0.8910	0.8745	0.8484	0.8122
3	0.8847	0.8910	1.0000	0.8828	0.8656	0.8383
4	0.8606	0.8745	0.8828	1.0000	0.8762	0.8588
5	0.8265	0.8484	0.8656	0.8762	1.0000	0.8719
6	0.7824	0.8122	0.8383	0.8588	0.8719	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	254290	37895	6.71	<.0001
UN(2,1)	ID	-12982	3620.52	-3.59	0.0003
UN(2,2)	ID	2346.46	551.40	4.26	<.0001
Session	ID	25934	1827.00	14.20	<.0001

Fit Statistics

-2 Res Log Likelihood	8333.4
AIC (smaller is better)	8341.4
AICC (smaller is better)	8341.4
BIC (smaller is better)	8351.8

Is the random slope significant? How do we know?

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
3	855.10	<.0001

Information Criteria

Neg2LogLike	**Parms**	AIC	AICC	HQIC	BIC	CAIC
8333.4	**4**	8341.4	8341.4	8345.6	8351.8	8355.8

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1961.89	52.6735	109	37.25	<.0001
Slope16	-163.64	20.8345	467	-7.85	<.0001
Slope26	130.75	23.2213	403	5.63	<.0001

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Session 1 Mean	1961.89	52.6735	109	37.25	<.0001
Session 2 Mean	1798.25	49.3756	107	36.42	<.0001
Session 3 Mean	1765.36	46.6212	103	37.87	<.0001
Session 4 Mean	1732.46	44.7989	100	38.67	<.0001
Session 5 Mean	1699.57	44.0246	100	38.60	<.0001
Session 6 Mean	1666.68	44.3533	102	37.58	<.0001
Slope from session 2-6	-32.8932	6.9936	163	-4.70	<.0001

What kind of individual differences in change does this model predict?

Model 5c: Random Slope, Random Deviation Slope Model

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Slope16}_{ti}\right) + \beta_{2i}\left(\text{Slope26}_{ti}\right) + e_{ti}$$

$$\text{Level 2: Intercept: } \beta_{0i} = \gamma_{00} + U_{0i}$$

$$\text{Slope12: } \beta_{1i} = \gamma_{10} + U_{1i}$$

$$\text{Slope26: } \beta_{2i} = \gamma_{20} + U_{2i}$$

```
TITLE1 "Model 5c: Random Slope, Random Deviation Slope Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = Slope16 Slope26 / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT Slope16 Slope26 / G GCORR V VCORR TYPE=UN SUBJECT=ID;
  REPEATED session / R TYPE=VC SUBJECT=ID;
  ESTIMATE "Session 1 Mean" Intercept 1 Slope16 0 Slope26 0;
  ESTIMATE "Session 2 Mean" Intercept 1 Slope16 1 Slope26 0;
  ESTIMATE "Session 3 Mean" Intercept 1 Slope16 2 Slope26 1;
  ESTIMATE "Session 4 Mean" Intercept 1 Slope16 3 Slope26 2;
  ESTIMATE "Session 5 Mean" Intercept 1 Slope16 4 Slope26 3;
  ESTIMATE "Session 6 Mean" Intercept 1 Slope16 5 Slope26 4;
  ESTIMATE "Slope from session 2-6" Slope16 1 Slope26 1;
RUN;
```

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9188.48345679	
1	1	**8275.37431715**	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	17673					
2		17673				
3			17673			
4				17673		
5					17673	
6						17673

Estimated G Matrix

Row	Effect	Person ID	Col1	Col2	Col3
1	Intercept	101	284312	-54270	43626
2	Slope16	101	-54270	63954	-65626
3	Slope26	101	43626	-65626	69916

Estimated G Correlation Matrix

Row	Effect	Person ID	Col1	Col2	Col3
1	Intercept	101	1.0000	-0.4025	0.3094
2	Slope16	101	-0.4025	1.0000	-0.9814
3	Slope26	101	0.3094	-0.9814	1.0000

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**301985**	230042	219399	208755	198111	187467
2	230042	**257400**	227410	215094	202778	190462
3	219399	227410	**235385**	208013	198314	188615
4	208755	215094	208013	**218604**	193850	186768
5	198111	202778	198314	193850	**207059**	184921
6	187467	190462	188615	186768	184921	**200747**

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8251	0.8229	0.8125	0.7923	0.7614
2	0.8251	1.0000	0.9239	0.9068	0.8784	0.8379
3	0.8229	0.9239	1.0000	0.9170	0.8983	0.8677

4	0.8125	0.9068	0.9170	1.0000	0.9111	0.8916
5	0.7923	0.8784	0.8983	0.9111	1.0000	0.9070
6	0.7614	0.8379	0.8677	0.8916	0.9070	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	284312	42731	6.65	<.0001
UN(2,1)	ID	-54270	18230	-2.98	0.0029
UN(2,2)	ID	63954	13244	4.83	<.0001
UN(3,1)	ID	43626	19049	2.29	0.0220
UN(3,2)	ID	-65626	14154	-4.64	<.0001
UN(3,3)	ID	69916	15434	4.53	<.0001
Session	ID	17673	1435.84	12.31	<.0001

Fit Statistics

-2 Res Log Likelihood	8275.4
AIC (smaller is better)	8289.4
AICC (smaller is better)	8289.6
BIC (smaller is better)	8307.7

Is the random deviation slope significant? How do we know?

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
6	913.11	<.0001

Information Criteria

Neg2LogLike	**Parms**	AIC	AICC	HQIC	BIC	CAIC
8275.4	**7**	8289.4	8289.6	8296.8	8307.7	8314.7

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1961.89	54.6805	100	35.88	<.0001
Slope16	-163.64	30.2188	100	-5.42	<.0001
Slope26	130.75	32.5530	100	4.02	0.0001

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Session 1 Mean	1961.89	54.6805	100	35.88	<.0001
Session 2 Mean	1798.25	49.7847	100	36.12	<.0001
Session 3 Mean	1765.36	46.9899	100	37.57	<.0001
Session 4 Mean	1732.46	44.9935	100	38.50	<.0001
Session 5 Mean	1699.57	43.9044	100	38.71	<.0001
Session 6 Mean	1666.68	43.7905	100	38.06	<.0001
Slope from session 2-6	-32.8932	6.5888	100	-4.99	<.0001

What kind of individual differences in change does this model predict?

Random Effect 95% CI = fixed effect $\pm \left(1.96*\sqrt{\text{Random Variance}}\right)$

Intercept 95% CI = $\gamma_{00} \pm \left(1.96*\sqrt{\tau^2_{U_0}}\right) \rightarrow$ $1{,}961.9 \pm \left(1.96*\sqrt{284{,}312}\right) = 917$ to $3{,}007$

Slope16 95% CI = $\gamma_{10} \pm \left(1.96*\sqrt{\tau^2_{U_1}}\right) \rightarrow$ $-163.6 \pm \left(1.96*\sqrt{63{,}954}\right) = -659$ to 322

Slope26 95% CI = $\gamma_{20} \pm \left(1.96*\sqrt{\tau^2_{U_2}}\right) \rightarrow$ $130.8 \pm \left(1.96*\sqrt{69{,}916}\right) = -338$ to 649

So how did we do? Let's compare model predictions in terms of means (top) and variances (bottom)?