Polynomial Models for Practice Effects in Number Match 3 Response Times

The models for this example come from Hoffman (in preparation) chapter 6. We will be examining change in response time (RT) in milliseconds over six practice sessions to a measure of processing speed (as measured by the number match 3 test) in a sample of 101 older adults. Of interest is the extent to which individual differences in change in RT can be described by polynomial models. Later we will examine alternative families of change (piecewise slopes and exponential models) as well.

SAS Syntax for Data Manipulation:

```
* Location for original SAS files for these models - change this path to your computer;
%LET example = F:\Example Data\Chapter 6 Data\CHAP Burst 1 RT;
LIBNAME example "&example.";

* Defining macro variable for datafile name to be replaced in code below;
%LET datafile=Example6;

* Bringing data into work library;
* Centering time at different points for polynomial models;
DATA &datafile.; SET example.&datafile.;
       c1sess = Session - 1; LABEL c1sess = "c1sess: Session Centered at 1";
       c6sess = Session - 6; LABEL c6sess = "c6sess: Session Centered at 6";
RUN;
```

Model 0: Saturated Means, Unstructured Variance Model (the answer key best baseline model)

```
TITLE1 "Model 0: Saturated Means, Unstructured Variances Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
       CLASS ID session;
       MODEL nm3rt = session / SOLUTION DDFM=Satterthwaite;
       REPEATED session / R RCORR TYPE=UN SUBJECT=ID;
       LSMEANS session /;
RUN;
```

The variable of *session* will be our categorical indicator for time—it will structure the **R** matrix via the REPEATED and CLASS statements given that we have balanced data. Here, *session* also appears on the MODEL statement so that we can estimate all possible mean differences across sessions.

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9155.43252939	
1	1	**8229.78846855**	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**301985**	235659	217994	202607	192154	195360
2	235659	**259150**	230217	213232	202092	193268
3	217994	230217	**233368**	205209	196919	188604
4	202607	213232	205209	**217544**	193676	185321
5	192154	202092	196919	193676	**212098**	187840
6	195360	193268	188604	185321	187840	**196733**

This **R matrix** UN structure lets all variances and covariances be what they want. THIS IS THE DATA we are trying to duplicate with our model for the variances.

Estimated R Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8424	0.8212	0.7905	0.7593	0.8015
2	0.8424	1.0000	0.9361	0.8981	0.8620	0.8559
3	0.8212	0.9361	1.0000	0.9108	0.8851	0.8802
4	0.7905	0.8981	0.9108	1.0000	0.9016	0.8958
5	0.7593	0.8620	0.8851	0.9016	1.0000	0.9196
6	0.8015	0.8559	0.8802	0.8958	0.9196	1.0000

Fit Statistics	
-2 Res Log Likelihood	8229.8
AIC (smaller is better)	8271.8
AICC (smaller is better)	8273.4
BIC (smaller is better)	8326.7

In REML, only parameters in the model for the variance count. Model comparisons in REML must have the same fixed effects in the model for the means.

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8229.8	21	8271.8	8273.4	8294.0	8326.7	8347.7

Solution for Fixed Effects

Effect	Session (1-6)	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		1672.14	44.1345	100	37.89	<.0001
Session	1	289.76	32.7000	100	8.86	<.0001
Session	2	143.04	26.2031	100	5.46	<.0001
Session	3	77.8986	22.8842	100	3.40	0.0010
Session	4	45.6604	20.7853	100	2.20	0.0303
Session	5	35.0397	18.1168	100	1.93	0.0559
Session	6	0	.	.	.	.

What do the effects of sessions 1-5 tell us?

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Session	5	100	16.72	<.0001

What does this F-test tell us?

Least Squares Means

Effect	Session (1-6)	Estimate	Standard Error	DF	t Value	Pr > |t|
Session	1	1961.89	54.6805	100	35.88	<.0001
Session	2	1815.17	50.6541	100	35.83	<.0001
Session	3	1750.03	48.0684	100	36.41	<.0001
Session	4	1717.80	46.4101	100	37.01	<.0001
Session	5	1707.18	45.8255	100	37.25	<.0001
Session	6	1672.14	44.1345	100	37.89	<.0001

The **saturated means model** lets all session means be what they want. THIS IS THE DATA we are trying to duplicate with our model for the means.

Model 1b: Empty Means, Random Intercept Model (the worst baseline model)

```
TITLE1 "Model 1b: Empty Means, Random Intercept Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
       CLASS ID session;
       MODEL nm3rt = / SOLUTION DDFM=Satterthwaite;
       RANDOM INTERCEPT / G V VCORR TYPE=UN SUBJECT=ID;
       REPEATED session / R TYPE=VC SUBJECT=ID;
RUN;
```

Level 1: $y_{ti} = \beta_{0i} + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9228.60509392	
1	1	**8536.86086232**	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	44900					
2		44900				
3			44900			
4				44900		
5					44900	
6						44900

This **R matrix** VC structure (equal variance over time, no covariance of any kind) will be used repeatedly as we add fixed and random effects of time to the model.

Estimated G Matrix

Row	Effect	Person ID	Col1
1	Intercept	101	200883

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**245783**	200883	200883	200883	200883	200883
2	200883	**245783**	200883	200883	200883	200883
3	200883	200883	**245783**	200883	200883	200883
4	200883	200883	200883	**245783**	200883	200883
5	200883	200883	200883	200883	**245783**	200883
6	200883	200883	200883	200883	200883	**245783**

This random intercept model predicts a compound symmetry pattern for the **V** matrix (equal variance, equal covariance over time).

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8173	0.8173	0.8173	0.8173	0.8173
2	0.8173	1.0000	0.8173	0.8173	0.8173	0.8173
3	0.8173	0.8173	1.0000	0.8173	0.8173	0.8173
4	0.8173	0.8173	0.8173	1.0000	0.8173	0.8173
5	0.8173	0.8173	0.8173	0.8173	1.0000	0.8173
6	0.8173	0.8173	0.8173	0.8173	0.8173	1.0000

This translates into equal correlation over time…

This correlation gets a special name of…?

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
UN(1,1)	ID	200883	29471	6.82	<.0001
Session	ID	44900	2825.63	15.89	<.0001

Fit Statistics

-2 Res Log Likelihood	8536.9
AIC (smaller is better)	8540.9
AICC (smaller is better)	8540.9
BIC (smaller is better)	8546.1

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	691.74	<.0001

This is the test of whether we need the random intercept variance, and thus whether the ICC is significantly > 0.

Information Criteria

Neg2LogLike	**Parms**	AIC	AICC	HQIC	BIC	CAIC
8536.9	**2**	8540.9	8540.9	8543.0	8546.1	8548.1

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1770.70	45.4206	100	38.98	<.0001

The empty means part of this model predicts no change on average (RT across six sessions is constant at 1770).

Model 2a: Fixed Linear Time, Random Intercept Model

```
TITLE1 "Model 2a: Fixed Linear, Random Intercept Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = c1sess / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT / G V VCORR TYPE=UN SUBJECT=ID;
  REPEATED session / R TYPE=VC SUBJECT=ID;

* Predicting means from model for each session;
  ESTIMATE "Session 1 Predicted Mean" intercept 1 c1sess 0;
  ESTIMATE "Session 2 Predicted Mean" intercept 1 c1sess 1;
  ESTIMATE "Session 3 Predicted Mean" intercept 1 c1sess 2;
  ESTIMATE "Session 4 Predicted Mean" intercept 1 c1sess 3;
  ESTIMATE "Session 5 Predicted Mean" intercept 1 c1sess 4;
  ESTIMATE "Session 6 Predicted Mean" intercept 1 c1sess 5;
RUN;
```

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Linear Time: $\beta_{1i} = \gamma_{10}$

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9202.30103061	
1	1	**8414.68804514**	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	35662					
2		35662				
3			35662			
4				35662		
5					35662	
6						35662

Estimated G Matrix

Row	Effect	Person ID	Col1
1	Intercept	101	202422

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	238084	202422	202422	202422	202422	202422
2	202422	238084	202422	202422	202422	202422
3	202422	202422	238084	202422	202422	202422
4	202422	202422	202422	238084	202422	202422
5	202422	202422	202422	202422	238084	202422
6	202422	202422	202422	202422	202422	238084

The predicted **V** matrix still has a compound symmetry pattern because we have not yet added to the model for the variance (still a random intercept variance only in **G**).

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8502	0.8502	0.8502	0.8502	0.8502
2	0.8502	1.0000	0.8502	0.8502	0.8502	0.8502
3	0.8502	0.8502	1.0000	0.8502	0.8502	0.8502
4	0.8502	0.8502	0.8502	1.0000	0.8502	0.8502
5	0.8502	0.8502	0.8502	0.8502	1.0000	0.8502
6	0.8502	0.8502	0.8502	0.8502	0.8502	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
UN(1,1)	ID	202422	29470	6.87	<.0001
Session	ID	35662	2246.48	15.87	<.0001

```
           Fit Statistics
-2 Res Log Likelihood          8414.7
AIC (smaller is better)        8418.7
AICC (smaller is better)       8418.7
BIC (smaller is better)        8423.9

  Null Model Likelihood Ratio Test
    DF    Chi-Square      Pr > ChiSq
     1        787.61          <.0001
```

Is the fixed linear time slope significant? How do we know?

```
                          Information Criteria
Neg2LogLike    Parms        AIC       AICC       HQIC        BIC       CAIC
     8414.7        2     8418.7     8418.7     8420.8     8423.9     8425.9

                  Solution for Fixed Effects
                        Standard
Effect       Estimate       Error      DF    t Value    Pr > |t|
Intercept     1899.63     46.7882     113      40.60      <.0001
c1sess       -51.5719      4.4918     504     -11.48      <.0001

                                  Estimates
                                      Standard
Label                        Estimate       Error      DF    t Value    Pr > |t|
Session 1 Predicted Mean      1899.63     46.7882     113      40.60      <.0001
Session 2 Predicted Mean      1848.06     45.9176     104      40.25      <.0001
Session 3 Predicted Mean      1796.49     45.4761     100      39.50      <.0001
Session 4 Predicted Mean      1744.92     45.4761     100      38.37      <.0001
Session 5 Predicted Mean      1693.34     45.9176     104      36.88      <.0001
Session 6 Predicted Mean      1641.77     46.7882     113      35.09      <.0001
```

What kind of change does this model predict?

What kind of individual differences in change does this model predict?

Model 2b: Random Linear Time Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Linear Time: $\beta_{1i} = \gamma_{10} + U_{1i}$

```
TITLE1 "Model 2b: Random Linear Time Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = c1sess / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT c1sess / G V VCORR TYPE=UN SUBJECT=ID;
  REPEATED session / R TYPE=VC SUBJECT=ID;
* Predicting means from model for each session;
  ESTIMATE "Session 1 Predicted Mean" intercept 1 c1sess 0;
  ESTIMATE "Session 2 Predicted Mean" intercept 1 c1sess 1;
  ESTIMATE "Session 3 Predicted Mean" intercept 1 c1sess 2;
  ESTIMATE "Session 4 Predicted Mean" intercept 1 c1sess 3;
  ESTIMATE "Session 5 Predicted Mean" intercept 1 c1sess 4;
  ESTIMATE "Session 6 Predicted Mean" intercept 1 c1sess 5;
RUN;
```

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9202.30103061	
1	1	**8372.10246085**	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	27905					
2		27905				
3			27905			
4				27905		
5					27905	
6						27905

Estimated G Matrix

Row	Effect	Person ID	Col1	Col2
1	Intercept	101	253258	-12701
2	c1sess	101	-12701	2233.83

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**281163**	240557	227856	215155	202455	189754
2	240557	**257995**	219623	209156	198689	188222
3	227856	219623	**239295**	203157	194924	186691
4	215155	209156	203157	**225063**	191158	185159
5	202455	198689	194924	191158	**215298**	183627
6	189754	188222	186691	185159	183627	**210001**

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8932	0.8784	0.8553	0.8229	0.7809
2	0.8932	1.0000	0.8839	0.8680	0.8430	0.8086
3	0.8784	0.8839	1.0000	0.8754	0.8588	0.8328
4	0.8553	0.8680	0.8754	1.0000	0.8684	0.8517
5	0.8229	0.8430	0.8588	0.8684	1.0000	0.8636
6	0.7809	0.8086	0.8328	0.8517	0.8636	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	253258	37897	6.68	<.0001
UN(2,1)	ID	-12701	3621.98	-3.51	0.0005
UN(2,2)	ID	2233.83	552.92	4.04	<.0001
Session	ID	27905	1963.42	14.21	<.0001

Fit Statistics

-2 Res Log Likelihood	8372.1
AIC (smaller is better)	8380.1
AICC (smaller is better)	8380.2
BIC (smaller is better)	8390.6

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
3	830.20	<.0001

Is the random linear time slope significant? How do we know?

Information Criteria

Neg2LogLike	**Parms**	AIC	AICC	HQIC	BIC	CAIC
8372.1	**4**	8380.1	8380.2	8384.3	8390.6	8394.6

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1899.63	51.4998	100	36.89	<.0001
c1sess	-51.5719	6.1567	100	-8.38	<.0001

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Session 1 Predicted Mean	1899.63	51.4998	100	36.89	<.0001
Session 2 Predicted Mean	1848.06	48.5766	100	38.04	<.0001
Session 3 Predicted Mean	1796.49	46.2922	100	38.81	<.0001
Session 4 Predicted Mean	1744.92	44.7443	100	39.00	<.0001
Session 5 Predicted Mean	1693.34	44.0107	100	38.48	<.0001
Session 6 Predicted Mean	1641.77	44.1322	100	37.20	<.0001

What kind of <u>average change</u> does this model predict?

What kind of <u>individual differences in change</u> does this model predict?

Model 3a: Fixed Quadratic, Random Linear Time Model

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + \beta_{2i}(\text{Time}_{ti})^2 + e_{ti}$$

$$\text{Level 2: Intercept: } \beta_{0i} = \gamma_{00} + U_{0i}$$

$$\text{Linear Time: } \beta_{1i} = \gamma_{10} + U_{1i}$$

$$\text{Quadratic Time: } \beta_{2i} = \gamma_{20}$$

```
TITLE1 "Model 3a: Fixed Quadratic, Random Linear Time Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = c1sess c1sess*c1sess / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT c1sess / G V VCORR TYPE=UN SUBJECT=ID;
  REPEATED session / R TYPE=VC SUBJECT=ID;
* Predicting means from model for each session;
  ESTIMATE "Session 1 Predicted Mean" intercept 1 c1sess 0 c1sess*c1sess 0;
  ESTIMATE "Session 2 Predicted Mean" intercept 1 c1sess 1 c1sess*c1sess 1;
  ESTIMATE "Session 3 Predicted Mean" intercept 1 c1sess 2 c1sess*c1sess 4;
  ESTIMATE "Session 4 Predicted Mean" intercept 1 c1sess 3 c1sess*c1sess 9;
  ESTIMATE "Session 5 Predicted Mean" intercept 1 c1sess 4 c1sess*c1sess 16;
  ESTIMATE "Session 6 Predicted Mean" intercept 1 c1sess 5 c1sess*c1sess 25;
* Predicting linear rate of change at each session (linear changes by 2*quad);
  ESTIMATE "Linear Slope at Session 1"  c1sess 1   c1sess*c1sess 0;
  ESTIMATE "Linear Slope at Session 2"  c1sess 1   c1sess*c1sess 2;
  ESTIMATE "Linear Slope at Session 3"  c1sess 1   c1sess*c1sess 4;
  ESTIMATE "Linear Slope at Session 4"  c1sess 1   c1sess*c1sess 6;
  ESTIMATE "Linear Slope at Session 5"  c1sess 1   c1sess*c1sess 8;
  ESTIMATE "Linear Slope at Session 6"  c1sess 1   c1sess*c1sess 10;
RUN;
```

Because twice the quadratic slope is how the linear slope changes per unit time, the value for *c1sess* used in estimating the linear slope per session gets multiplied by 2.

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9193.25780414	
1	1	8341.47727191	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	26176					
2		26176				
3			26176			
4				26176		
5					26176	
6						26176

Estimated G Matrix

Row	Effect	Person ID	Col1	Col2
1	Intercept	101	254164	-12948
2	c1sess	101	-12948	2332.67

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**280339**	241216	228268	215320	202372	189424
2	241216	**256776**	219985	209370	198755	188140
3	228268	219985	**237879**	203420	195138	186855
4	215320	209370	203420	**223646**	191521	185571
5	202372	198755	195138	191521	**214079**	184286
6	189424	188140	186855	185571	184286	**209178**

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8991	0.8839	0.8599	0.8261	0.7822
2	0.8991	1.0000	0.8901	0.8737	0.8477	0.8118
3	0.8839	0.8901	1.0000	0.8819	0.8647	0.8377
4	0.8599	0.8737	0.8819	1.0000	0.8753	0.8580
5	0.8261	0.8477	0.8647	0.8753	1.0000	0.8709
6	0.7822	0.8118	0.8377	0.8580	0.8709	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	254164	37896	6.71	<.0001
UN(2,1)	ID	-12948	3620.70	-3.58	0.0003
UN(2,2)	ID	2332.67	551.58	4.23	<.0001
Session	ID	26176	1844.01	14.20	<.0001

Fit Statistics

-2 Res Log Likelihood	8341.5
AIC (smaller is better)	8349.5
AICC (smaller is better)	8349.5
BIC (smaller is better)	8359.9

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
3	851.78	<.0001

Is the fixed quadratic time slope significant? How do we know?

Information Criteria

Neg2LogLike	**Parms**	AIC	AICC	HQIC	BIC	CAIC
8341.5	**4**	8349.5	8349.5	8353.7	8359.9	8363.9

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1945.85	52.2433	106	37.25	<.0001
c1sess	-120.90	14.5415	502	-8.31	<.0001
c1sess*c1sess	13.8656	2.6348	403	5.26	<.0001

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Session 1 Predicted Mean	1945.85	52.2433	106	37.25	<.0001
Session 2 Predicted Mean	1838.82	48.6084	100	37.83	<.0001
Session 3 Predicted Mean	1759.51	46.8223	105	37.58	<.0001
Session 4 Predicted Mean	1707.94	45.2925	105	37.71	<.0001
Session 5 Predicted Mean	1684.10	44.0458	100	38.24	<.0001
Session 6 Predicted Mean	1687.99	44.9976	108	37.51	<.0001
Linear Slope at Session 1	-120.90	14.5415	502	-8.31	<.0001
Linear Slope at Session 2	-93.1687	10.0191	419	-9.30	<.0001
Linear Slope at Session 3	-65.4375	6.6968	139	-9.77	<.0001
Linear Slope at Session 4	-37.7062	6.6968	139	-5.63	<.0001
Linear Slope at Session 5	-9.9750	10.0191	419	-1.00	0.3200
Linear Slope at Session 6	17.7562	14.5415	502	1.22	0.2226

What happens to the linear slope over sessions because of the quadratic slope?

Model 3b: Random Quadratic Time Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + \beta_{2i}(\text{Time}_{ti})^2 + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Linear Time: $\beta_{1i} = \gamma_{10} + U_{1i}$

Quadratic Time: $\beta_{2i} = \gamma_{20} + U_{2i}$

```
TITLE1 "Model 3b: Random Quadratic Model";
PROC MIXED DATA=&datafile. NOCLPRINT COVTEST IC METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = c1sess c1sess*c1sess / SOLUTION DDFM=Satterthwaite;
  RANDOM INTERCEPT c1sess c1sess*c1sess / G V VCORR TYPE=UN SUBJECT=ID;
  REPEATED session / R TYPE=VC SUBJECT=ID;
* Predicting means from model for each session;
  ESTIMATE "Session 1 Predicted Mean" intercept 1 c1sess 0 c1sess*c1sess 0;
  ESTIMATE "Session 2 Predicted Mean" intercept 1 c1sess 1 c1sess*c1sess 1;
  ESTIMATE "Session 3 Predicted Mean" intercept 1 c1sess 2 c1sess*c1sess 4;
  ESTIMATE "Session 4 Predicted Mean" intercept 1 c1sess 3 c1sess*c1sess 9;
  ESTIMATE "Session 5 Predicted Mean" intercept 1 c1sess 4 c1sess*c1sess 16;
  ESTIMATE "Session 6 Predicted Mean" intercept 1 c1sess 5 c1sess*c1sess 25;
* Predicting linear rate of change at each session (linear changes by 2*quad);
  ESTIMATE "Linear Slope at Session 1"  c1sess 1   c1sess*c1sess 0;
  ESTIMATE "Linear Slope at Session 2"  c1sess 1   c1sess*c1sess 2;
  ESTIMATE "Linear Slope at Session 3"  c1sess 1   c1sess*c1sess 4;
  ESTIMATE "Linear Slope at Session 4"  c1sess 1   c1sess*c1sess 6;
  ESTIMATE "Linear Slope at Session 5"  c1sess 1   c1sess*c1sess 8;
  ESTIMATE "Linear Slope at Session 6"  c1sess 1   c1sess*c1sess 10;
RUN;
```

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	9193.25780414	
1	1	**8302.74566856**	0.00000000

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	20298					
2		20298				
3			20298			
4				20298		
5					20298	
6						20298

Estimated G Matrix

Row	Effect	Person ID	Col1	Col2	Col3
1	Intercept	101	276206	-35734	3901.96
2	c1sess	101	-35734	25840	-3903.32
3	c1sess*c1sess	101	3901.96	-3903.32	634.47

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**296504**	244374	220346	204122	195702	195085
2	244374	**251508**	219312	208680	199315	191215
3	220346	219312	**235842**	209043	199808	187840
4	204122	208680	209043	**225508**	197182	184958
5	195702	199315	199808	197182	**211735**	182571
6	195085	191215	187840	184958	182571	**200977**

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8949	0.8333	0.7894	0.7811	0.7992
2	0.8949	1.0000	0.9005	0.8762	0.8637	0.8505
3	0.8333	0.9005	1.0000	0.9064	0.8941	0.8628
4	0.7894	0.8762	0.9064	1.0000	0.9024	0.8688
5	0.7811	0.8637	0.8941	0.9024	1.0000	0.8850
6	0.7992	0.8505	0.8628	0.8688	0.8850	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	276206	41442	6.66	<.0001
UN(2,1)	ID	-35734	11941	-2.99	0.0028
UN(2,2)	ID	25840	5864.41	4.41	<.0001
UN(3,1)	ID	3901.96	1949.06	2.00	0.0453
UN(3,2)	ID	-3903.32	982.61	-3.97	<.0001
UN(3,3)	ID	634.47	172.37	3.68	0.0001
Session	ID	20298	1649.11	12.31	<.0001

Fit Statistics

-2 Res Log Likelihood	8302.7
AIC (smaller is better)	8316.7
AICC (smaller is better)	8316.9
BIC (smaller is better)	8335.1

Is the random quadratic time slope significant? How do we know?

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
6	890.51	<.0001

Information Criteria

Neg2LogLike	**Parms**	AIC	AICC	HQIC	BIC	CAIC
8302.7	**7**	8316.7	8316.9	8324.2	8335.1	8342.1

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1945.85	53.8497	100	36.13	<.0001
c1sess	-120.90	20.0476	100	-6.03	<.0001
c1sess*c1sess	13.8656	3.4154	100	4.06	<.0001

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Session 1 Predicted Mean	1945.85	53.8497	100	36.13	<.0001
Session 2 Predicted Mean	1838.82	48.4864	100	37.92	<.0001
Session 3 Predicted Mean	1759.51	46.9973	100	37.44	<.0001
Session 4 Predicted Mean	1707.94	45.8959	100	37.21	<.0001
Session 5 Predicted Mean	1684.10	44.2395	100	38.07	<.0001
Session 6 Predicted Mean	1687.99	44.2038	100	38.19	<.0001
Linear Slope at Session 1	-120.90	20.0476	100	-6.03	<.0001
Linear Slope at Session 2	-93.1687	13.6497	100	-6.83	<.0001
Linear Slope at Session 3	-65.4375	8.0028	100	-8.18	<.0001
Linear Slope at Session 4	-37.7062	5.9242	100	-6.36	<.0001
Linear Slope at Session 5	-9.9750	9.9733	100	-1.00	0.3196
Linear Slope at Session 6	17.7562	16.0362	100	1.11	0.2708

Computing random effects confidence intervals for each random effect:

$$\text{Random Effect 95\% CI} = \text{fixed effect} \pm \left(1.96*\sqrt{\text{Random Variance}}\right)$$

$$\text{Intercept 95\% CI} = \gamma_{00} \pm \left(1.96*\sqrt{\tau^2_{U_0}}\right) \rightarrow 1{,}945.9 \pm \left(1.96*\sqrt{276{,}209}\right) = 916 \text{ to } 2{,}976$$

$$\text{Linear Time Slope 95\% CI} = \gamma_{10} \pm \left(1.96*\sqrt{\tau^2_{U_1}}\right) \rightarrow -120.9 \pm \left(1.96*\sqrt{25{,}840}\right) = -436 \text{ to } 194$$

$$\text{Quadratic Time Slope 95\% CI} = \gamma_{20} \pm \left(1.96*\sqrt{\tau^2_{U_2}}\right) \rightarrow 13.9 \pm \left(1.96*\sqrt{634}\right) = -36 \text{ to } 63$$

Is it a problem that the CIs for the slopes overlap 0? What does this mean?

Last thing: is a VC R matrix (equal residual variance, no residual covariance) sufficient?

```
TITLE1 "Testing AR1 Residual Correlation Across Time";
PROC MIXED DATA=&datafile. NOCLPRINT NOITPRINT COVTEST METHOD=REML;
       CLASS ID session;
       MODEL nm3rt = c1sess c1sess*c1sess / SOLUTION DDFM=Satterthwaite;
       RANDOM INTERCEPT c1sess c1sess*c1sess / G V VCORR TYPE=UN SUBJECT=ID;
       REPEATED session / R RCORR TYPE=AR(1) SUBJECT=ID; RUN;
```

Estimated R Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.1711	0.02927	0.005008	0.000857	0.000147
2	0.1711	1.0000	0.1711	0.02927	0.005008	0.000857
3	0.02927	0.1711	1.0000	0.1711	0.02927	0.005008
4	0.005008	0.02927	0.1711	1.0000	0.1711	0.02927
5	0.000857	0.005008	0.02927	0.1711	1.0000	0.1711
6	0.000147	0.000857	0.005008	0.02927	0.1711	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	272126	41822	6.51	<.0001
UN(2,1)	ID	-35004	12302	-2.85	0.0044
UN(2,2)	ID	24064	6386.03	3.77	<.0001
UN(3,1)	ID	3941.63	1996.01	1.97	0.0483
UN(3,2)	ID	-3554.21	1076.64	-3.30	0.0010
UN(3,3)	ID	547.63	194.97	2.81	0.0025
AR(1)	**ID**	**0.1711**	**0.1286**	**1.33**	**0.1833**
Residual		24534	4585.43	5.35	<.0001

Is the AR1 residual correlation significant? How do we know?

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8300.7	**8**	8316.7	8316.9	8325.1	8337.6	8345.6

```
TITLE1 "Testing Lag-1 Toeplitz Residual Covariance Across Time";
PROC MIXED DATA=&datafile. NOCLPRINT NOITPRINT COVTEST METHOD=REML;
       CLASS ID session;
       MODEL nm3rt = c1sess c1sess*c1sess / SOLUTION DDFM=Satterthwaite;
       RANDOM INTERCEPT c1sess c1sess*c1sess / G V VCORR TYPE=UN SUBJECT=ID;
       REPEATED session / R RCORR TYPE=TOEP(2) SUBJECT=ID; RUN;
```

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	23059	2856.63				
2	2856.63	23059	2856.63			
3		2856.63	23059	2856.63		
4			2856.63	23059	2856.63	
5				2856.63	23059	2856.63
6					2856.63	23059

Estimated R Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.1239				
2	0.1239	1.0000	0.1239			
3		0.1239	1.0000	0.1239		
4			0.1239	1.0000	0.1239	
5				0.1239	1.0000	0.1239
6					0.1239	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	273602	41647	6.57	<.0001
UN(2,1)	ID	-35273	12222	-2.89	0.0039
UN(2,2)	ID	24667	6180.41	3.99	<.0001
UN(3,1)	ID	3929.32	1987.62	1.98	0.0481
UN(3,2)	ID	-3664.04	1038.33	-3.53	0.0004
UN(3,3)	ID	573.36	184.45	3.11	0.0009
TOEP(2)	**ID**	**2856.63**	**2130.45**	**1.34**	**0.1800**
Residual		23059	2867.62	8.04	<.0001

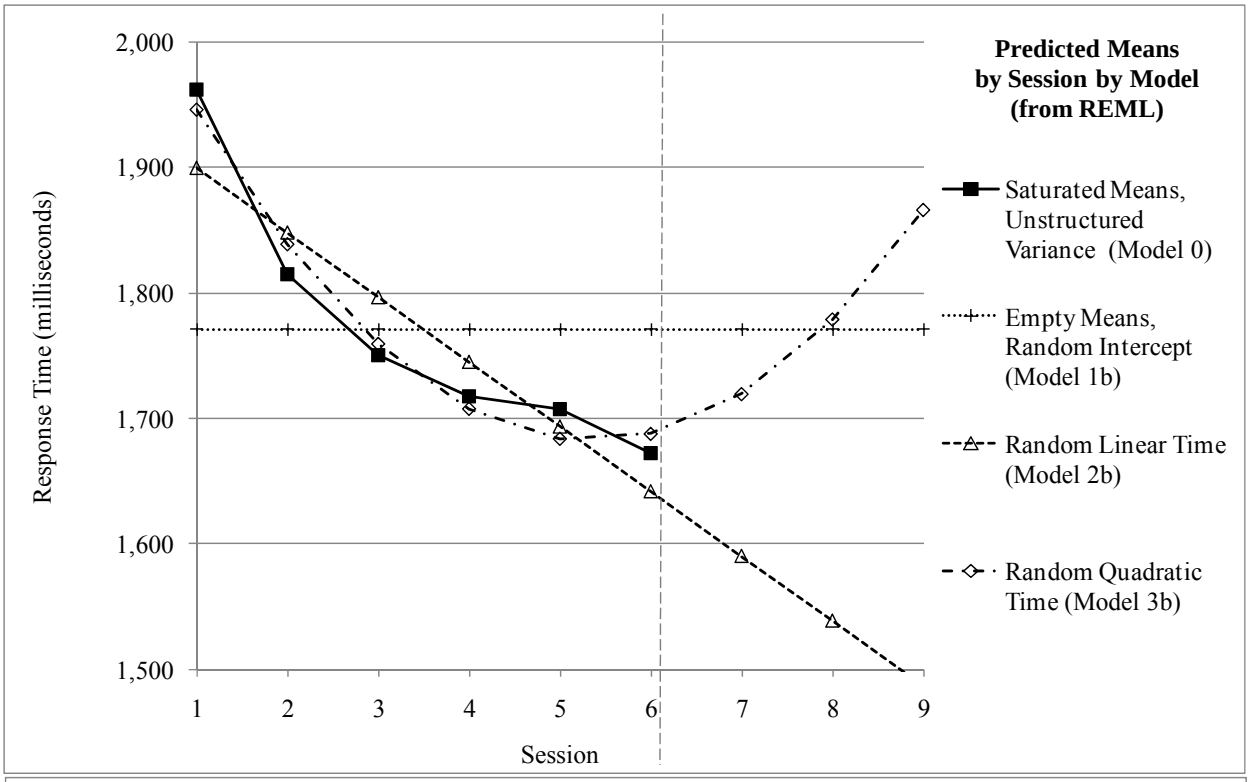
Is the lag-1 Toeplitz residual covariance significant? How do we know?

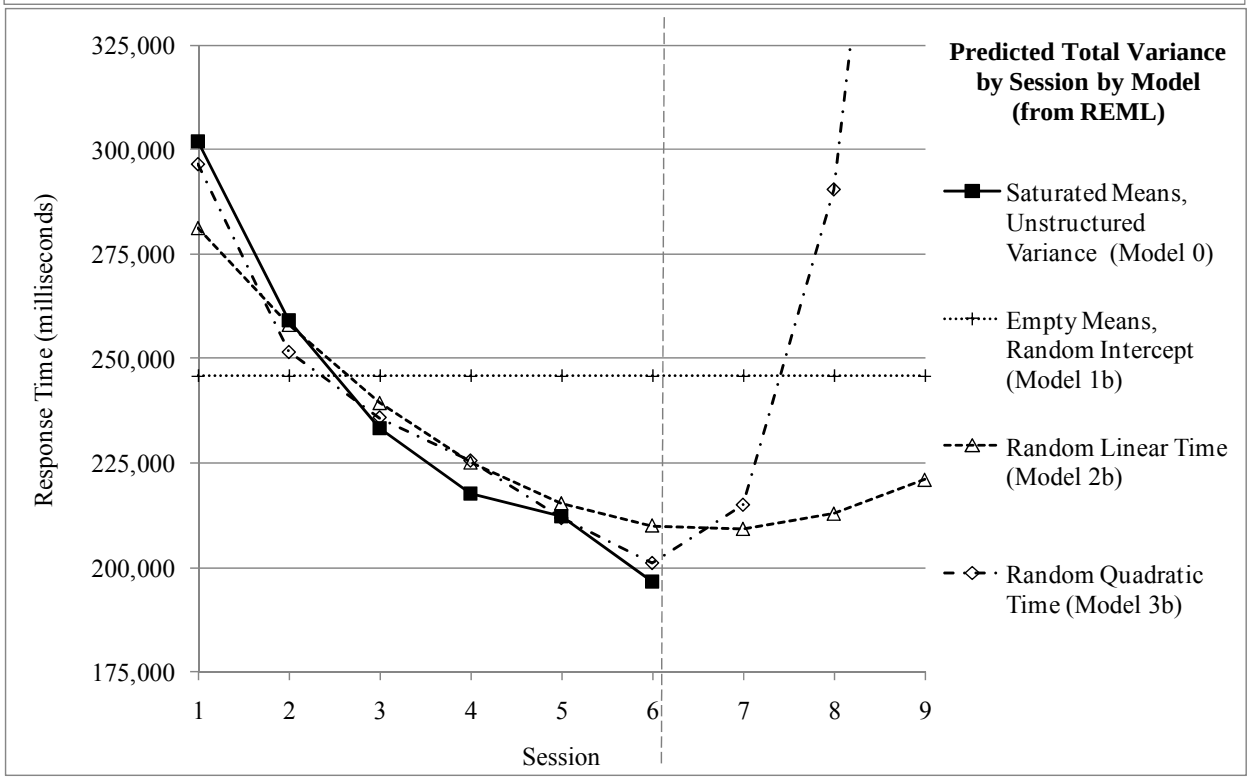
Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8300.9	**8**	8316.9	8317.1	8325.4	8337.8	8345.8

Given that neither of these **R** matrix additions improved model fit, I'm calling us done (for now).

So how did we do? Let's compare model predictions in terms of means (top) and variances (bottom)?





What happens to the model parameters if we pick a different centering point?

Fixed Quadratic, Random Linear Time (Session 1 = Time 0)

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	254164	37896	6.71	<.0001
UN(2,1)	ID	-12948	3620.70	-3.58	0.0003
UN(2,2)	ID	2332.67	551.58	4.23	<.0001
Session	ID	26176	1844.01	14.20	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1945.85	52.2433	106	37.25	<.0001
C1sess	-120.90	14.5415	502	-8.31	<.0001
C1sess*C1sess	13.8656	2.6348	403	5.26	<.0001

Fixed Quadratic, Random Linear Time (Session 6 = Time 0)

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	183002	27836	6.57	<.0001
UN(2,1)	ID	-1284.53	2767.79	-0.46	0.6426
UN(2,2)	ID	2332.67	551.58	4.23	<.0001
Session	ID	26176	1844.01	14.20	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1687.99	44.9976	108	37.51	<.0001
C6sess	17.7562	14.5415	502	1.22	0.2226
C6sess*C6sess	13.8656	2.6348	403	5.26	<.0001

Which parameters change and why?

Random Quadratic Time (Session 1 = Time 0)

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	276206	41442	6.66	<.0001
UN(2,1)	ID	-35734	11941	-2.99	0.0028
UN(2,2)	ID	25840	5864.41	4.41	<.0001
UN(3,1)	ID	3901.96	1949.06	2.00	0.0453
UN(3,2)	ID	-3903.32	982.61	-3.97	<.0001
UN(3,3)	ID	634.47	172.37	3.68	0.0001
Session	ID	20298	1649.11	12.31	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1945.85	53.8497	100	36.13	<.0001
C1sess	-120.90	20.0476	100	-6.03	<.0001
C1sess*C1sess	13.8656	3.4154	100	4.06	<.0001

Random Quadratic Time (Session 6 = Time 0)

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	180678	27943	6.47	<.0001
UN(2,1)	ID	-1645.67	7298.42	-0.23	0.8216
UN(2,2)	ID	11221	3863.75	2.90	0.0018
UN(3,1)	ID	247.15	1545.72	0.16	0.8730
UN(3,2)	ID	2441.39	788.06	3.10	0.0019
UN(3,3)	ID	634.47	172.37	3.68	0.0001
Session	ID	20298	1649.11	12.31	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1687.99	44.2038	100	38.19	<.0001
C6sess	17.7562	16.0362	100	1.11	0.2708
C6sess*C6sess	13.8656	3.4154	100	4.06	<.0001

Which parameters change and why?

Parameter	Fixed Linear Session=1	Random Linear Session=1	Fixed Quadratic Session=1	Fixed Quadratic Session=6	Random Quadratic Session=1	Random Quadratic Session=6
Fixed Effects:						
Intercept	1899.6	1899.6	1945.9	1688.0	1945.9	1679.0
Linear	-51.6	-51.6	-120.9	17.8	-120.9	17.8
Quadratic			13.9	13.9	13.9	13.9
Variance Components:						
Residual Variance	35662.0	27905.0	26176.0	26176.0	20298.0	20298.0
Intercept Variance	202422.0	253258.0	254164.0	183002.0	276206.0	180678.0
Linear Variance		2233.8	2332.7	2332.7	25840.0	11221.0
Quadratic Variance					634.5	634.5
Intercept-Linear Covariance		-12701.0	-12948.0	-1284.5	-35734.0	-1645.7
Intercept-Quadratic Covariance					3902.0	247.2
Linear-Quadratic Covariance					-3903.3	2441.4
Model Fit:						
REML Deviance	8414.7	8372.1	8341.5	8341.5	8302.7	8302.7
AIC	8418.7	8380.1	8349.5	8349.5	8316.7	8316.7
BIC	8423.9	8390.6	8359.9	8359.9	8335.1	8335.1
Total Number of Parameters	4	6	7	7	10	10

95% Random Effects CIs for Random Quadratic (Session 1 = 0):

Intercept: 1946 ± [1.96*SQRT(276206)] = 916 to 2976
95% of sample predicted to have *initial* RTs from 916 to 2976…

Linear: −120.9 ± [1.96*SQRT(25840)] = −436 to 194
… and linear slopes *(at session 1)* from −436 to 194

Quad: 13.9 ± [1.96*SQRT(634)] = −36 to 63
… and quadratic slopes from -36 and 63

95% Random Effects CIs for Random Quadratic (Session 6 = 0):

Intercept: 1679 ± [1.96*SQRT(180678)] = 846 to 2512
95% of sample predicted to have *final* RTs from 846 to 2512…

Linear: 17.8 ± [1.96*SQRT(11224)] = −190 to 225
… and linear slopes *(at session 6)* to −190 to 225

Quad: 13.9 ± [1.96*SQRT(634)] = −36 to 63
… and quadratic slopes from −36 and 63