

Example 3a: From Between-Person to Within-Person Models for Longitudinal Data (complete syntax, data, and output available for SAS, STATA, and R electronically)

The models for this example come from Hoffman (2015) chapter 3 example 3a. We will be examining the extent to which a learning achievement outcome can be predicted from group (control as the reference vs. treatment) and time (pre-test as the reference vs. post-test) in a sample of 50 children. For an example results section, please see the end of chapter 3.

SAS Syntax for Data Import and Manipulation:

```
* Defining global variable for file location to be replaced in code below;
%LET filesave=C:\Dropbox\21_PSQF7375_Longitudinal\PSQF7375_Longitudinal_Example3a;
* Location for SAS files for these models (uses macro variable filesave);
LIBNAME filesave "&filesave.";

* Import and stack chapter 3 two-occasion multivariate data;
* Create new variable on left from old variable on right, OUTPUT writes data;
DATA work.Chapter3a; SET filesave.SAS_Chapter3a;
time=1; outcome=outcome1; OUTPUT;
time=2; outcome=outcome2; OUTPUT;
DROP outcome1 outcome2;
LABEL time = "time: Occasion (1=pre-test, 2=post-test)"
outcome = "outcome: Learning Outcome"; RUN;

* Center predictors for analysis;
DATA work.Chapter3a; SET work.Chapter3a;
time1 = time - 1; treat = group - 1;
LABEL time1 = "time1: Time (0=pre-test, 1=post-test)"
treat = "treat: Treatment Group (0=control, 1=treatment)"; RUN;
```

STATA Syntax for Data Import and Manipulation:

```
// Defining global variable for file location to be replaced in code below
global filesave "C:\Dropbox\21_PSQF7375_Longitudinal\PSQF7375_Longitudinal_Example3a"

// Import and stack chapter 3a two-occasion multivariate data
// List time-varying variables first, i(level2ID) j(newtimeID)
use "$filesave\STATA_Chapter3a.dta", clear
reshape long outcome, i(personid) j(time)
label variable time "time: Occasion (1=pre-test, 2=post-test)"
label variable outcome "outcome: Learning Outcome"

// Center predictors for analysis
gen time1 = time - 1
gen treat = group - 1
label variable time1 "time1: Time (0=pre-test, 1=post-test)"
label variable treat "treat: Treatment Group (0=control, 1=treatment)"
```

R Syntax for Data Import and Manipulation:

```
# Define variables for working directory and data name
filesave = "C:\\Dropbox\\21_PSQF7375_Longitudinal\\PSQF7375_Longitudinal_Example3a/"
filename = "SAS_Chapter3a.sas7bdat"
setwd(dir=filesave)

# Import chapter 3 two-occasion multivariate data with labels
Example3a_wide = read_sas(data_file=paste0(filesave,filename))
# Convert to data frame as data frame without labels to use for analysis
Example3a_wide = as.data.frame(Example3a_wide)
# Labels can be used with gls, so I will use them here
# Stack into long format (one row per occasion per person), add labels, and sort
Example3a = reshape(Example3a_wide,
  varying=c("outcome1","outcome2"),
  v.names="outcome",
  idvar="PersonID",
  timevar="time",
  times=c(1,2),
  direction="long")
```

```

Example3a = apply_labels(data=Example3a,
  time = "time: Occasion (1=pre-test, 2=post-test)",
  outcome = "outcome: Learning Outcome")
Example3a = sort_asc(Example3a, PersonID, time)

# Center predictors for analysis
Example3a$time1 = Example3a$time-1
Example3a$treat = Example3a$group-1
Example3a = apply_labels(Example3a,
  time1 = "time1: Time (0=pre-test, 1=post-test)",
  treat = "treat: Treatment Group (0=control, 1=treatment)")

```

SAS, STATA, and R Syntax for Descriptive Statistics:

```

* CLASS= means per group and time, WAYS= means overall=0, per category=1, per cell=2;
TITLE1 "Chapter 3a Example: Means by group and time for learning outcome";
PROC MEANS NDEC=2 MEAN STDERR MIN MAX DATA=work.Chapter3a;
  CLASS group time; WAYS 0 1 2; VAR outcome;
RUN; TITLE1;

display "Chapter 3a Example: Means by group and time for learning outcome"
tabulate group time, summarize(outcome)

print("Chapter 3a Example: Means by group and time for learning outcome")
describeBY(x=Example3a$outcome, digits=2,
  group=list(group=Example3a$group, time=Example3a$time))

```

SAS Output (that created the means in slide 15 in Lecture 3):

N	Mean	Std Error	Minimum	Maximum
100	53.34	0.64	37.53	68.62

time: Occasion (1=pre-test, 2=post-test)		N	Mean	Std Error	Minimum	Maximum
1	50	49.92	0.73	37.53	62.13	
2	50	56.76	0.79	44.56	68.62	

group: Treatment Group (1=control, 2=treatment)		N	Mean	Std Error	Minimum	Maximum
1	50	51.99	0.89	37.53	67.11	
2	50	54.69	0.87	40.53	68.62	

group: Treatment Group (1=control, 2=treatment)		time: Occasion (1=pre-test, 2=post-test)		N	Mean	Std Error	Minimum	Maximum
1	1	25	49.08	1.14	37.53	59.55		
		25	54.90	1.13	44.56	67.11		
2	1	25	50.76	0.91	40.53	62.13		
		25	58.62	0.99	47.43	68.62		

SAS, STATA, and R Syntax for 3.1: Between-Person Empty Model $y_{ti} = \beta_0 + e_{ti}$

```
TITLE "Eq 3.1: Empty Between-Person model via SAS MIXED";
PROC MIXED DATA=work.Chapter3a NOCLPRINT COVTEST METHOD=REML;
  CLASS PersonID time;
  MODEL outcome = / SOLUTION DDFM=BW;
  REPEATED time / R RCORR TYPE=VC SUBJECT=PersonID;
RUN; TITLE;
```

The pattern of variances and covariances over time (= ID variable) is controlled by SAS REPEATED, STATA RESIDUALS, and R CORRELATION.

STATA MIXED includes a random intercept by default, removed by NOCONSTANT.

```
display "Eq 3a.1: Empty Between-Person Model via STATA MIXED"
mixed outcome , ///
  || personid: , noconstant variance reml ///
  residuals(independent,t(time)) dfmethod(residual) dftable(pvalue) ,
  estat ic, n(50) , // Information criteria using level-2 N
```

```
print("Eq 3.1: Empty Between-Person model via R GLS (no correlation, =LM)")
empty_BP = gls(data=Example3a, method="REML",
  model=outcome~1,
  correlation=NULL) # VC R matrix (no correlation)
print("Show results with total original variance")
summary(empty_BP); summary(empty_BP)$sigma^2
```

SAS Output:

Dimensions	
Covariance Parameters	1
Columns in X	1
Columns in Z	0
Subjects	50
Max Obs Per Subject	2

This table tells you how many parameters are in your model for the means (“columns in x”, the fixed effects, or 1 fixed intercept here) and in your model for the variance (“covariance parameters”, or 1 residual variance here). It also tells you how many observations were read per subject, as defined by SUBJECT= on the REPEATED line.

Estimated R Matrix
for PersonID 1

Row	Col1	Col2
1	40.3353	
2		40.3353

Estimated R Correlation
Matrix for PersonID 1

Row	Col1	Col2
1	1.0000	
2		1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
time	PersonID	40.3353	5.7330	7.04	<.0001

This is the estimate of the residual variance σ_e^2 . It is labeled “time” because that is how the R matrix is structured via the REPEATED line.

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
0	0.00	1.0000

This “null model” LRT examines the need for any random effects variances and covariances. Because we don’t have any (yet), df = 0.

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
651.6	1	653.6	653.6	654.3	655.5	656.5

In REML using SAS, model df = # for calculating AIC and BIC only includes parameters in the model for the variance.

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > t
Intercept	53.3396	0.6351	49	83.99	<.0001

This is the estimate of the fixed intercept β_0 .

SAS, STATA, and R Syntax for 3.2: Within-Person Empty Model $y_{ti} = \beta_0 + U_{0i} + e_{ti}$

```

TITLE "Eq 3.2: Empty Within-Person model via SAS MIXED";
PROC MIXED DATA=work.Chapter3a NOCLPRINT COVTEST METHOD=REML;
  CLASS PersonID time;
  MODEL outcome = / SOLUTION DDFM=BW;
  REPEATED time / R RCORR TYPE=CS SUBJECT=PersonID; RUN; TITLE;

display "Eq 3a.2: Empty Within-Person Model via STATA MIXED"
mixed outcome , ///
  || personid: , noconstant variance reml ///
  residuals(exchangeable,t(time)) dfmethod(repeated) dftable(pvalue) ,
  estat ic, n(50) , // Information criteria using level-2 N
  estat wcorrelation, covariance // RCOV matrix
  estat wcorrelation // RCORR matrix

print("Eq 3.2: Empty Within-Person model via R GLS (CS correlation)")
empty_WP = gls(data=Example3a, method="REML",
  model=outcome~1,
  correlation=corCompSymm(form=~1|PersonID) )
print("Show results with total original variance")
summary(empty_WP); summary(empty_WP)$sigma^2
print("Show RCOV and RCORR matrices")
getVarCov(empty_WP); corMatrix(empty_WP$modelStruct$corStruct)[[5]]
print("Show likelihood ratio test comparing variance model fit (stay tuned)")
anova(empty_WP,empty_BP)
  
```

SAS Output:

	Dimensions	
Covariance Parameters		2
Columns in X		1
Columns in Z		0
Subjects		50
Max Obs Per Subject		2

We still have 1 fixed effect, the fixed intercept, but now the model for the variance includes random intercept variance and residual variance.

Estimated R Matrix
for PersonID 1

Row	Col1	Col2	$\begin{bmatrix} \sigma_e^2 + \tau_{u_0}^2 & \tau_{u_0}^2 \\ \tau_{u_0}^2 & \sigma_e^2 + \tau_{u_0}^2 \end{bmatrix}$
1	40.4590	12.2526	
2	12.2526	40.4590	

Estimated R Correlation
Matrix for PersonID 1

Row	Col1	Col2	$\begin{bmatrix} 1 & ICC \\ ICC & 1 \end{bmatrix}$
1	1.0000	0.3028	
2	0.3028	1.0000	

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z	Pr > Z
CS	PersonID	12.2526	6.0256	2.03	0.0420
Residual		28.2064	5.6413	5.00	<.0001

CS = Random Intercept Variance $\tau_{U_0}^2$
Residual = Residual Variance σ_e^2

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	4.77	0.0289

Now we have a random intercept variance, so df=1. This is the LRT model comparison of BP vs. WP. Who wins?

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
646.8	2	650.8	650.9	652.3	654.6	656.6

Now the model for the variance df=2.

Solution for Fixed Effects
Standard

Effect	Estimate	Error	DF	t Value	Pr > t
Intercept	53.3396	0.7260	49	73.47	<.0001

This is still the estimate of the fixed intercept β_0 , but note the SE differs.

Which is the better empty model, and how do you know?

What is the ICC for these data and what does it mean?

SAS, STATA, and R Syntax and SAS Output for 3.7 (top): Between-Person Conditional Model

$$y_{ti} = \beta_0 + \beta_1(\text{Time}_{ti} - 1) + \beta_2(\text{Treat}_i) + \beta_3(\text{Time}_{ti} - 1)(\text{Treat}_i) + e_{ti}$$

Control at Pre-Test: $\hat{y}_{ti} = \beta_0 + \beta_1(0) + \beta_2(0) + \beta_3(0)(0)$

Control at Post-Test: $\hat{y}_{ti} = \beta_0 + \beta_1(1) + \beta_2(0) + \beta_3(0)(0)$

Treatment at Pre-Test: $\hat{y}_{ti} = \beta_0 + \beta_1(0) + \beta_2(1) + \beta_3(0)(0)$

Treatment at Post-Test: $\hat{y}_{ti} = \beta_0 + \beta_1(1) + \beta_2(1) + \beta_3(1)(1)$

Time Effect for Control: $\beta_1 + \beta_3(0)$

Time Effect for Treatment: $\beta_1 + \beta_3(1)$

Group Effect for Pre-Test: $\beta_2 + \beta_3(0)$

Group Effect for Post-Test: $\beta_2 + \beta_3(1)$

```
TITLE1 "Eq 3.7 (top): Between-Person Conditional (Predictor) Model via SAS MIXED";
TITLE2 "Not using CLASS statement, manually dummy coding group and time";
PROC MIXED DATA=work.Chapter3a NOCLPRINT COVTEST METHOD=REML;
  CLASS PersonID time;
  MODEL outcome = time1 treat time1*treat / SOLUTION DDFM=BW;
  REPEATED time / R RCORR TYPE=VC SUBJECT=PersonID;
ESTIMATE "Mean: Control Group at Pre-Test"      intercept 1 time1 0 treat 0 time1*treat 0;
ESTIMATE "Mean: Control Group at Post-Test"     intercept 1 time1 1 treat 0 time1*treat 0;
ESTIMATE "Mean: Treatment Group at Pre-Test"    intercept 1 time1 0 treat 1 time1*treat 0;
ESTIMATE "Mean: Treatment Group at Post-Test"   intercept 1 time1 1 treat 1 time1*treat 1;
ESTIMATE "Time Effect for Control Group"        time1 1 time1*treat 0;
ESTIMATE "Time Effect for Treatment Group"      time1 1 time1*treat 1;
ESTIMATE "Group Effect at Pre-Test"             treat 1 time1*treat 0;
ESTIMATE "Group Effect at Post-Test"           treat 1 time1*treat 1;
ESTIMATE "Time*Group Interaction Effect"        time1*treat 1;
CONTRAST "DF=3 F-test for Model Fixed Effects" time1 1, treat 1, time1*treat 1 / CHISQ;
RUN; TITLE1; TITLE2;
```

```
display "Eq 3.7 (top): Between-Person Conditional (Predictor) Model via STATA MIXED"
display "Not using i., manually dummy coding group and time"
mixed outcome c.time1 c.treat c.time1#c.treat, ///
  || personid: , noconstant variance reml ///
  residuals(independent,t(time)) dfmethod(residual) dftable(pvalue),
estat ic, n(50), // Information criteria using level-2 N
// Mean: Control Group at Pre-Test
lincom _cons*1 + c.time1*0 + c.treat*0 + c.time1#c.treat*0, small
// Mean: Control Group at Post-Test
lincom _cons*1 + c.time1*1 + c.treat*0 + c.time1#c.treat*0, small
// Mean: Treatment Group at Pre-Test
lincom _cons*1 + c.time1*0 + c.treat*1 + c.time1#c.treat*0, small
// Mean: Treatment Group at Post-Test
lincom _cons*1 + c.time1*1 + c.treat*1 + c.time1#c.treat*1, small
```

```
// Time Effect for Control Group
lincom c.time1*1 + c.time1#c.treat*0, small
// Time Effect for Treatment Group
lincom c.time1*1 + c.time1#c.treat*1, small
// Group Effect at Pre-Test
lincom c.treat*1 + c.time1#c.treat*0, small
// Group Effect at Post-Test
lincom c.treat*1 + c.time1#c.treat*1, small
// Time*Group Interaction
lincom c.treat*1 + c.time1#c.treat*1, small
// DF=3 F-test for Model Fixed Effects
test (c.time1=0) (c.treat=0) (c.time1#c.treat=0), small

print("Eq 3.7 (top): Between-Person Conditional (Predictor) Model via R GLS (no correlation, =LM)")
print("Not using factor variables, manually dummy coding group and time")
cond_BP = gls(data=Example3a, method="REML",
              model=outcome~1+time1+treat+time1:treat,
              correlation=NULL)
print("Show results with total leftover variance")
summary(cond_BP); summary(cond_BP)$sigma^2
print("Get linear combination estimates, add correct denominator DF, no adjustment needed")
summary(ghst(model=cond_BP, df=48, test=adjusted("none"), linfoct=rbind(
  "Mean: Control Group at Pre-Test"      =c(1,0,0,0),
  "Mean: Control Group at Post-Test"     =c(1,1,0,0),
  "Mean: Treatment Group at Pre-Test"    =c(1,0,1,0),
  "Mean: Treatment Group at Post-Test"   =c(1,1,1,1),
  "Time Effect for Control Group"       =c(0,1,0,0),
  "Time Effect for Treatment Group"     =c(0,1,0,1),
  "Group Effect at Pre-Test"           =c(0,0,1,0),
  "Group Effect at Post-Test"          =c(0,0,1,1),
  "Time*Group Interaction"              =c(0,0,0,1))))
print("Get and show test of model R2 -- joint test of fixed effects -- using correct DF")
modelF_BP=ghst(model=cond_BP, linfoct=c("time1=0", "treat=0", "time1:treat=0"), df=48)
BP=summary(modelF_BP, test=Ftest()); BP
print("Get and show hidden results for F, dfnum, dfden, and p-value")
BP$test$fstat; BP$test$df; BP$df
pf(BP$test$fstat, df1=BP$test$df, df2=BP$df, lower.tail=FALSE)
```

SAS Output:

```
Dimensions
Covariance Parameters      1
Columns in X               4
Columns in Z               0
Subjects                   50
Max Obs Per Subject       2
```

Now we have 4 parameters in the model for the means and 1 parameter in the model for the variance (σ_e^2).

```
Estimated R Matrix
for PersonID 1
Row      Col1      Col2
1      27.2245
2                      27.2245
```

```
Estimated R Correlation
Matrix for PersonID 1
Row      Col1      Col2
1      1.0000
2                      1.0000
```

```
Covariance Parameter Estimates
Cov      Standard      Z
Parm      Subject      Estimate      Error      Value      Pr > Z
time      PersonID      27.2245      3.9295      6.93      <.0001
```

This is the estimate of the residual variance σ_e^2 . It is labeled "time" because that is how the R matrix is structured via the REPEATED line.

```
Null Model Likelihood Ratio Test
DF      Chi-Square      Pr > ChiSq
0      0.00      1.0000
```

This "null model" LRT examines the need for any random effects variances and covariances. Because we don't have any (yet), $df = 0$.

		Information Criteria				
Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
602.5	1	604.5	604.5	605.2	606.4	607.4

BP Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > t	
Intercept	49.0768	1.0435	48	47.03	<.0001	beta0
time1	5.8224	1.4758	48	3.95	0.0003	beta1
treat	1.6819	1.4758	48	1.14	0.2601	beta2
time1*treat	2.0425	2.0871	48	0.98	0.3327	beta3

BP Model Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > t	
Mean: Control Group at Pre-Test	49.0768	1.0435	48	47.03	<.0001	
Mean: Control Group at Post-Test	54.8992	1.0435	48	52.61	<.0001	
Mean: Treatment Group at Pre-Test	50.7587	1.0435	48	48.64	<.0001	
Mean: Treatment Group at Post-Test	58.6236	1.0435	48	56.18	<.0001	
Time Effect for Control Group	5.8224	1.4758	48	3.95	0.0003	beta1
Time Effect for Treatment Group	7.8649	1.4758	48	5.33	<.0001	beta1 + beta3*1
Group Effect at Pre-Test	1.6819	1.4758	48	1.14	0.2601	beta2
Group Effect at Post-Test	3.7245	1.4758	48	2.52	0.0150	beta2 + beta3*1
Time*Group Interaction Effect	2.0425	2.0871	48	0.98	0.3327	beta3

BP Model Contrasts

Label	Num	Den	Chi-Square	F Value	Pr > ChiSq	Pr > F
DF=3 F-test for Model Fixed Effects	3	48	50.68	16.89	<.0001	<.0001

These results assume independent observations... what happens if that's not the case?

SAS, STATA, and R Syntax for 3.7 (bottom): Within-Person Conditional Model

$$y_{ti} = \beta_0 + \beta_1(\text{Time}_{ti} - 1) + \beta_2(\text{Treat}_i) + \beta_3(\text{Time}_{ti} - 1)(\text{Treat}_i) + U_{0i} + e_{ti}$$

```
TITLE1 "Eq 3.7 (bottom): Within-Person Conditional (Predictor) Model via SAS MIXED";
TITLE2 "Not using CLASS statement, manually dummy coding group and time";
PROC MIXED DATA=work.Chapter3a NOCLPRINT IC COVTEST METHOD=REML;
  CLASS PersonID time;
  MODEL outcome = time1 treat time1*treat / SOLUTION DDFM=BW;
  REPEATED time / R RCORR TYPE=CS SUBJECT=PersonID;
ESTIMATE "Mean: Control Group at Pre-Test"    intercept 1 time1 0 treat 0 time1*treat 0;
ESTIMATE "Mean: Control Group at Post-Test"   intercept 1 time1 1 treat 0 time1*treat 0;
ESTIMATE "Mean: Treatment Group at Pre-Test"  intercept 1 time1 0 treat 1 time1*treat 0;
ESTIMATE "Mean: Treatment Group at Post-Test" intercept 1 time1 1 treat 1 time1*treat 1;
ESTIMATE "Time Effect for Control Group"      time1 1 time1*treat 0;
ESTIMATE "Time Effect for Treatment Group"    time1 1 time1*treat 1;
ESTIMATE "Group Effect at Pre-Test"           treat 1 time1*treat 0;
ESTIMATE "Group Effect at Post-Test"         treat 1 time1*treat 1;
ESTIMATE "Time*Group Interaction Effect"      time1*treat 1;
CONTRAST "DF=3 F-test for Model Fixed Effects" time1 1, treat 1, time1*treat 1 / CHISQ;
RUN; TITLE1; TITLE2;
```

```
display "Eq 3.7 (bottom): Within-Person Conditional (Predictor) Model via STATA MIXED"
display "Not using i., manually dummy coding group and time"
mixed outcome c.time1 c.treat c.time1#c.treat, ///
  || personid: , noconstant variance reml ///
  residuals(exchangeable,t(time)) dfmethod(repeated) dftable(pvalue),
```

```

estat ic, n(50), // Information criteria using level-2 N
estat wcorrelation, covariance // RCOV matrix
estat wcorrelation // RCORR matrix
// Mean: Control Group at Pre-Test
lincom _cons*1 + c.time1*0 + c.treat*0 + c.time1#c.treat*0, small
// Mean: Control Group at Post-Test
lincom _cons*1 + c.time1*1 + c.treat*0 + c.time1#c.treat*0, small
// Mean: Treatment Group at Pre-Test
lincom _cons*1 + c.time1*0 + c.treat*1 + c.time1#c.treat*0, small
// Mean: Treatment Group at Post-Test
lincom _cons*1 + c.time1*1 + c.treat*1 + c.time1#c.treat*1, small
// Time Effect for Control Group
lincom c.time1*1 + c.time1#c.treat*0, small
// Time Effect for Treatment Group
lincom c.time1*1 + c.time1#c.treat*1, small
// Group Effect at Pre-Test
lincom c.treat*1 + c.time1#c.treat*0, small
// Group Effect at Post-Test
lincom c.treat*1 + c.time1#c.treat*1, small
// Time*Group Interaction
lincom c.treat*1 + c.time1#c.treat*1, small
// DF=3 F-test for Model Fixed Effects
test (c.time1=0)(c.treat=0)(c.time1#c.treat=0), small

print("Eq 3.7 (bottom): Within-Person Conditional (Predictor) Model via gls (CS correlation)")
print("Not using factor variables, manually dummy coding group and time")
cond_WP = gls(data=Example3a, method="REML",
              model=outcome~1+time1+treat+time1:treat,
              correlation=corCompSymm(form=~1|PersonID) )
print("Show results with total leftover variance")
summary(cond_WP); summary(cond_WP)$sigma^2
print("Show RCOV and RCORR matrices")
getVarCov(cond_WP); corMatrix(cond_WP$modelStruct$corStruct)[[5]]
print("Get linear combination estimates, add correct denominator DF, no adjustment needed")
summary(ghlt(model=cond_WP, df=48, test=adjusted("none"), linfct=rbind(
  "Mean: Control Group at Pre-Test" =c(1,0,0,0),
  "Mean: Control Group at Post-Test" =c(1,1,0,0),
  "Mean: Treatment Group at Pre-Test" =c(1,0,1,0),
  "Mean: Treatment Group at Post-Test" =c(1,1,1,1),
  "Time Effect for Control Group" =c(0,1,0,0),
  "Time Effect for Treatment Group" =c(0,1,0,1),
  "Group Effect at Pre-Test" =c(0,0,1,0),
  "Group Effect at Post-Test" =c(0,0,1,1),
  "Time*Group Interaction" =c(0,0,0,1))))
print("Get and show test of model R2 -- joint test of fixed effects -- using correct DF")
modelF_WP=ghlt(model=cond_WP, linfct=c("time1=0","treat=0","time1:treat=0"),df=48)
WP=summary(modelF_WP, test=Ftest()); WP # Print results
print("Get and show hidden results for F, dfnum, dfden, and p-value")
WP$test$Fstat; WP$test$df; WP$df
pf(WP$test$Fstat,df1=WP$test$df,df2=WP$df,lower.tail=FALSE)
print("Show likelihood ratio test comparing variance model fit (stay tuned)")
anova(empty_WP,empty_BP)

```

SAS Output:

	Dimensions
Covariance Parameters	2
Columns in X	4
Columns in Z	0
Subjects	50
Max Obs Per Subject	2

We still have 4 parameters in the model for the means, but now we have 2 parameters in the model for the variance ($\tau_{u_0}^2$ and σ_e^2).

Estimated R Matrix
for PersonID 1

Row	Col1	Col2
1	27.2245	22.7794
2	22.7794	27.2245

$$\begin{bmatrix} \sigma_e^2 + \tau_{u_0}^2 & \tau_{u_0}^2 \\ \tau_{u_0}^2 & \sigma_e^2 + \tau_{u_0}^2 \end{bmatrix}$$

Estimated R Correlation
Matrix for PersonID 1

Row	Col1	Col2	
1	1.0000	0.8367	$\begin{bmatrix} 1 & \text{ICC} \\ \text{ICC} & 1 \end{bmatrix}$
2	0.8367	1.0000	

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
CS	PersonID	22.7794	5.1236	4.45	<.0001
Residual		4.4451	0.9073	4.90	<.0001

CS = Random Intercept Variance $\tau_{U_0}^2$
Residual = Residual Variance σ_e^2

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	57.81	<.0001

Now we have a random intercept variance, so df=1. This is the LRT model comparison of conditional BP vs. WP. Who wins?

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
544.7	2	548.7	548.8	550.2	552.5	554.5

BP Solution for Fixed Effects (REPEATED FROM ABOVE FOR COMPARISON)

Effect	Estimate	Standard Error	DF	t Value	Pr > t	
Intercept	49.0768	1.0435	48	47.03	<.0001	beta0
time1	5.8224	1.4758	48	3.95	0.0003	beta1
treat	1.6819	1.4758	48	1.14	0.2601	beta2
time1*treat	2.0425	2.0871	48	0.98	0.3327	beta3

WP Solution for Fixed Effects

Which results differ from the BP model, and why?

Effect	Estimate	Standard Error	DF	t Value	Pr > t	
Intercept	49.0768	1.0435	48	47.03	<.0001	beta0
time1	5.8224	0.5963	48	9.76	<.0001	beta1
treat	1.6819	1.4758	48	1.14	0.2601	beta2
time1*treat	2.0425	0.8433	48	2.42	0.0193	beta3

WP Model Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > t	
Mean: Control Group at Pre-Test	49.0768	1.0435	48	47.03	<.0001	
Mean: Control Group at Post-Test	54.8992	1.0435	48	52.61	<.0001	
Mean: Treatment Group at Pre-Test	50.7587	1.0435	48	48.64	<.0001	
Mean: Treatment Group at Post-Test	58.6236	1.0435	48	56.18	<.0001	
Time Effect for Control Group	5.8224	0.5963	48	9.76	<.0001	beta1
Time Effect for Treatment Group	7.8649	0.5963	48	13.19	<.0001	beta1 + beta3*1
Group Effect at Pre-Test	1.6819	1.4758	48	1.14	0.2601	beta2
Group Effect at Post-Test	3.7245	1.4758	48	2.52	0.0150	beta2 + beta3*1
Time*Group Interaction Effect	2.0425	0.8433	48	2.42	0.0193	beta3

WP Model Contrasts

Label	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
DF=3 F-test for Model Fixed Effects	3	48	272.93	90.98	<.0001	<.0001

$$y_{ti} = \beta_0 + \beta_1(\text{Time}_{ti} - 1) + \beta_2(\text{Treat}_i) + \beta_3(\text{Time}_{ti} - 1)(\text{Treat}_i) + U_{0i} + e_{ti}$$

Think about person as a design factor (so person by time by group)...

What other terms that could possibly be included are missing? Are they really missing???